Massless particles

A relativistic concept !

Classically, a particle with no mass is nothing

$E_K = 0 \quad , \quad p = 0$

$F = 0 \qquad W = 0 \quad , \ldots\ldots$

But massless particles make perfect sense in relativity

$E = pc \qquad\qquad \text{for} \quad m = 0$

Furthermore $\dfrac{u}{c} = \dfrac{pc}{E} = 1$

$\Rightarrow$ massless particles move at speed of light

Note/ Be careful : our formulae were derived from

$$E = \frac{mc^2}{\sqrt{1-\frac{u^2}{c^2}}} \qquad p = \frac{mu}{\sqrt{1-\frac{u^2}{c^2}}}$$

As $u \to c$, $\dfrac{1}{\sqrt{1-\frac{u^2}{c^2}}} \to 0$, but precisely

because $m \to 0$, E & p are non-zero & finite

Examples of massless particles : photon , gluon , graviton (force carriers)

Note 2: Rest mass for massless particles is a misnomer because we cannot find a frame where they (such as photons) are at rest!

→ this amelleorate some conceptual difficulties (of classical physics) of objects with no mass and non-vanishing energy etc

Note 3: Rest mass of a system of photons can be non-zero

5 MeV 2 MeV

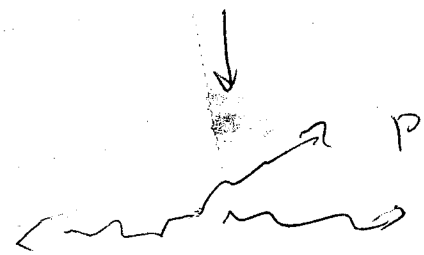
$$E_{total} = 5\text{ MeV} + 2\text{ MeV} = 7\text{ MeV}$$

$$p_{total} = \frac{5\text{ MeV} - 2\text{ MeV}}{c} = \frac{3\text{ MeV}}{c}$$

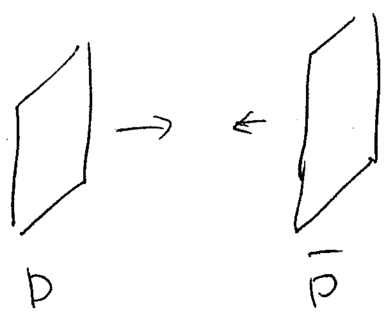
$$\Rightarrow mc^2 = \sqrt{7^2 - 3^2}\text{ MeV} = 6.3\text{ MeV}$$

Creation / Annihilation of Particles

e^- → ← e^+ same mass, opposite charge (Paul Dirac)



at least two
why? (HW)



annihilate → create universe

In annihilation processes above

mass → energy

$$E_{total} = E_1 + E_2$$

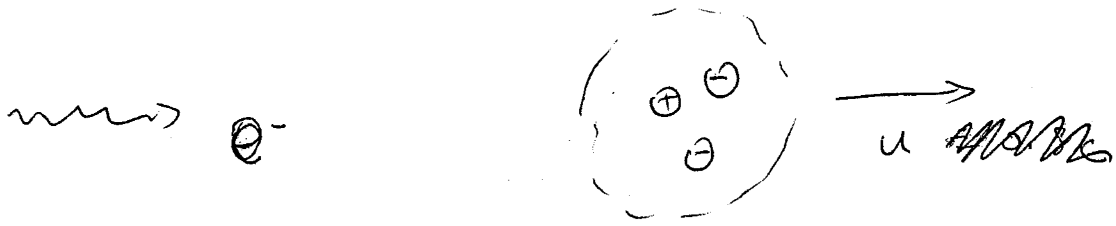
$$\simeq 2 \times 0.511 \text{ MeV} \quad \text{for } u \ll c$$

$$P_{total} = 0$$

$$\Rightarrow E_\gamma^{total} = 1.022 \text{ MeV} \quad \text{for sum of all photons}$$

If there are 2 photons, $E_\gamma = 0.511$ MeV for each γ & $p_\gamma = E_\gamma / c = \frac{0.511 \text{ MeV}}{c}$

Creation : reverse process



[Need electrons to be nearby originally, see HW Problem. $\gamma \to e^- e^+$ impossible]

Before $E_i = E_\gamma + mc^2$ $\qquad$ $E_f = E_i$

$$p_i = \frac{E_\gamma}{c} \qquad p_f = p_i$$

After: $Mc^2 = 3mc^2$

invariant mass of system since constituents are not moving wrt another

$$(3mc^2)^2 = E_f^2 - p_f^2 c^2$$

$$= E_i^2 - p_i^2 c^2$$

$$= (E_\gamma + mc^2)^2 - E_\gamma^2$$

$$= 2mc^2 E_\gamma + (mc^2)^2$$

$$\boxed{E_\gamma = 4mc^2} > 2mc^2$$

why we need more energy?

Because we use γ to create $e^- e^+$ pair and γ carries momentum $\Rightarrow$ $e^+ e^-$ are not at rest

$$E_K = E - 3mc^2 = (E_\gamma + mc^2) - 3mc^2$$

$$= 4mc^2 + mc^2 - 3mc^2 = \underbrace{2mc^2}$$

Kinetic energy shared by the system.

$$\frac{u}{c} = \frac{pc}{E} = \frac{\frac{E_\gamma}{c} \times c}{E_\gamma + mc^2} = \frac{4mc^2}{5mc^2} = 0.8$$

Approximations useful in simplifying calculations

Non-relativistic : $u \ll c$

$$\gamma = \frac{1}{\sqrt{1-\frac{u^2}{c^2}}} \simeq 1 + \frac{1}{2}\frac{u^2}{c^2} + \frac{3}{8}\frac{u^4}{c^4} + \cdots$$

$$E_k = (\gamma - 1) \simeq \frac{1}{2}mu^2 + \frac{3}{2}\frac{(\frac{1}{2}mu^2)^2}{mc^2}$$

$$\frac{E_k - \frac{1}{2}mu^2}{E_k} \simeq \frac{3}{2}\frac{E_k}{mc^2}$$

if $\frac{E_k}{mc^2} \sim 1\%$

$\Rightarrow$ error $\sim 1.5\%$

Relativistic $u \simeq c \Rightarrow \gamma \gg 1$

$$\gamma = \frac{1}{\sqrt{1-\frac{u^2}{c^2}}} \Rightarrow 1 - \frac{u^2}{c^2} = \frac{1}{\gamma^2}$$

$$\Rightarrow \frac{u}{c} = \sqrt{1 - \frac{1}{\gamma^2}}$$

$$\simeq 1 - \frac{1}{2\gamma^2} + \cdots$$

In other words $E \simeq pc$ ~~better~~ accurate to about 1% if $E \gtrsim 8mc^2$