

Chapter 12 Static Equilibrium and Elasticity

①

Stationary remains stationary \Rightarrow ~~static~~ ^{static} equilibrium
(not to be confused with stable equilibrium)

Why important? \Rightarrow e.g. building bridges

Conditions?

① Net force = 0 $\sum \vec{F}_i = 0$

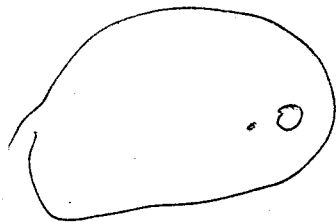
② Can still rotate

Net torque = 0 about any pt $\sum_i \vec{\tau}_i = 0$

\uparrow
choose ~~any~~ pivot point to simplify calculations!

Center of Gravity

Rigid body: composed of many particles

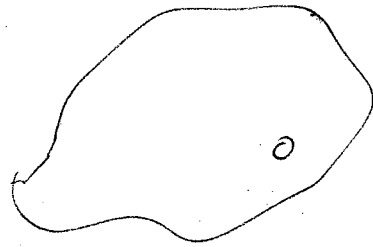


$$\vec{W} = \sum \vec{w}_i$$

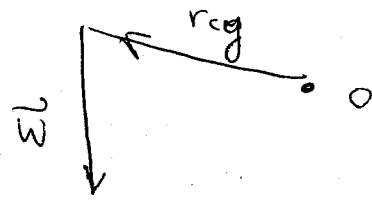
$$\vec{\tau}_i = \vec{r}_i \times \vec{w}_i \quad \text{torque about } O$$

$$\vec{\tau}_{\text{net}} = \sum_i \vec{r}_i \times \vec{w}_i$$

Center of gravity



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$$\vec{\tau}_{net} = \vec{r}_{cg} \times \vec{W}$$

\vec{r}_{cg} = position vector of center of gravity relative to \circ

For uniform gravitational field,

$$\vec{r}_{cg} = \vec{r}_{cm}$$

Proof:
$$\begin{aligned} \vec{\tau}_{net} &= \sum_i \vec{r}_i \times \vec{W}_i = \sum_i \vec{r}_i \times m_i \vec{g} \\ &= \sum_i m_i \vec{r}_i \times \vec{g} \\ &= \left(\sum_i m_i \vec{r}_i \right) \times \vec{g} \\ &= M \vec{r}_{cm} \times \vec{g} \\ &= \vec{r}_{cm} \times (M \vec{g}) = \vec{r}_{cm} \times \vec{W} \end{aligned}$$

$$\Rightarrow \vec{r}_{cg} = \vec{r}_{cm}$$

~~When O is directly above center of gravity~~ When O is directly above center of gravity



$$\vec{\tau}_{net} = \vec{r}_{cg} \times \vec{W} = 0 \Rightarrow \text{static equilibrium}$$