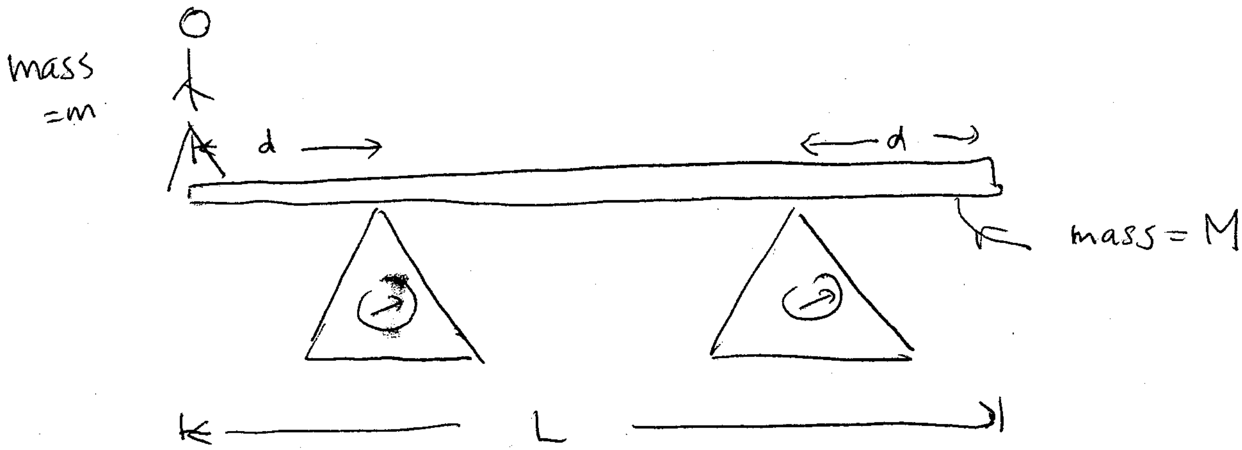
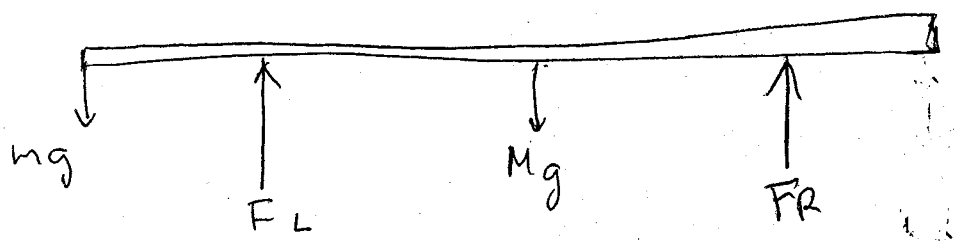
Examples



Static Equilibrium : what are the readings on the scale

Free body diagram



First: $\sum F_i = 0 \Rightarrow F_L + F_R - mg - Mg = 0$

Next: $\sum_i \tau_i = 0$ How do we choose the pivot point?

Pick a point where an unknown force is applied.

Torque about the pt of application of $\vec{F_L}$

Convention + counter clockwise

— clockwise

$$0 = F_R(L-2d) - Mg\left(\frac{L-2d}{2}\right) + mgd$$

$$\Rightarrow \boxed{F_R = \left(\frac{1}{2}M - \frac{md}{L-2d}\right)g}$$

Newton's law

$$F_L = (M+m)g - F_R$$

$$= \left(\frac{1}{2}M + \frac{m(L-d)}{L-2d}\right)g$$

$F_L > F_R$ as expected (why?)

If Reading on $F_R = 0$, what is mass?

$$\boxed{m = \frac{L-2d}{2d}M}$$

Check if $F_R = 0$. only torque $= mgd - Mg\frac{(L-2d)}{2}$

Other pivot point

Can choose point of application of F_R

What if we choose center of plank?

Newton's law stay the same $\qquad F_L + F_R - mg - Mg = 0 \qquad (1)$

Torque $\qquad F_R\left(\frac{L-2d}{2}\right) - F_L\left(\frac{L-2d}{2}\right) + mg\frac{L}{2} = 0 \qquad (2)$

From (1) $\qquad F_L + F_R = (M+m)g$

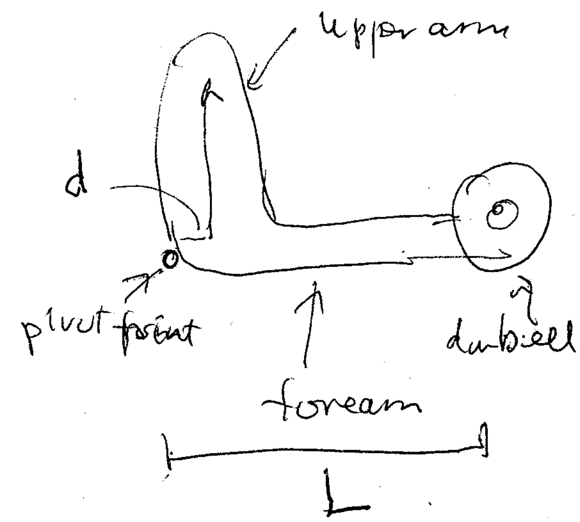
(2) $\qquad F_L - F_R = mg\left(\frac{L}{L-2d}\right)$

$$\Rightarrow \quad F_L = \left(\frac{1}{2}M + m\frac{(L-d)}{L-2d}\right)g$$

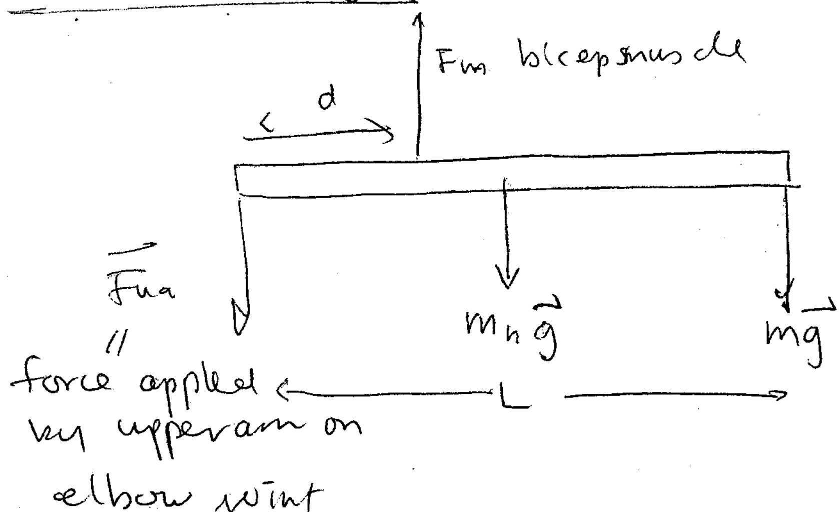
$$F_R = \left(\frac{1}{2}M - \frac{md}{L-2d}\right)g$$

$\Rightarrow$ Same result.

Example Weight lifting



Free body diagram



$$0 = \sum F_i \Rightarrow F_{ua,x} = 0$$

$$F_{ua,y} - F_m + m_h g + mg = 0$$

$$F_{ua,y} = F_m - (M + m_h) g$$

$\sum \tau = 0$ ~~...~~ Choose pivot pt : point of application of $\vec{F}_{ua}$

$$-m_h g \frac{L}{2} + F_m d - mgL = 0$$

$$F_m = \left(\frac{m_h}{2} + m\right) g \frac{L}{d}$$

$$F_{ua,y} = F_m - (m + m_h) g$$

$$= \left(\frac{m_h}{2}\frac{L}{d} + m\frac{L}{d} - m - m_h\right) g$$

$$= \left[m_h\left(\frac{L}{2d} - 1\right) + m\left(\frac{L}{d} - 1\right)\right] g$$

> 0 pointing down to keep forearm in equilibrium

~~...~~ Since $L \gg d$

$F_m \gg m$

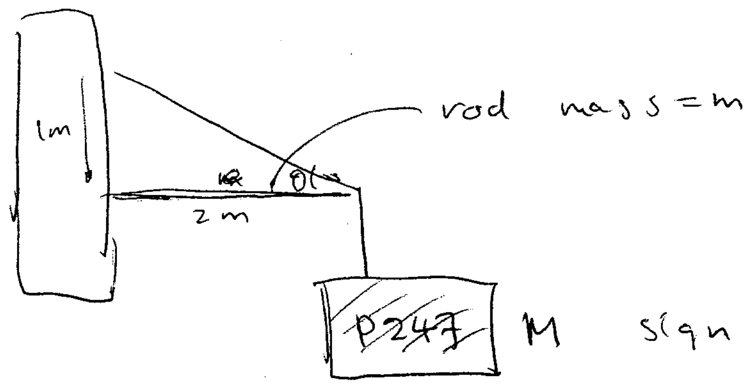
$F_{ua} \gg m$

Putting in some numbers:

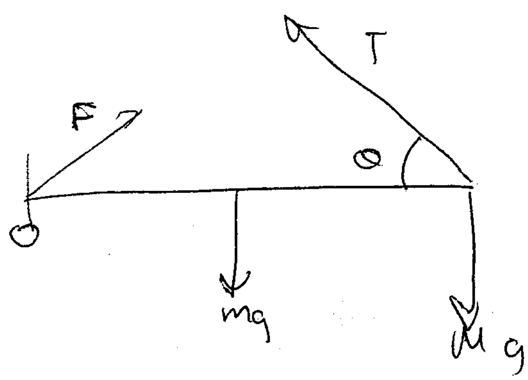
$L = 30\text{cm}$
$d = 3.4\text{cm}$
$m = 6\text{kg}$
$m_h = 1\text{kg}$

$\Rightarrow F_m = 563\text{N}$, $F_{ua} = 494\text{N}$ many times bigger than mg

Example: Hanging a sign



Free body diagram



$\tan\theta = \frac{1}{2}$

$\theta = 26.6^\circ$

Pick O as pivot point (why? we don't know the direction & magnitude of $\vec{F}$)

$$\Sigma \tau = 0 \Rightarrow T L \sin\theta - M g L - m g \frac{L}{2} = 0$$

$$\Rightarrow T = \frac{1}{\sin\theta}\left(M + \frac{m}{2}\right) g$$

independent of L

For $M = 20$ kg, $m = 4$ kg,

$$\Rightarrow T = 483 \text{N}$$

Force: $F_x + T_x = 0 \Rightarrow F_x = -T_x = +T\cos\theta$

$F_y + T_y - Mg - mg = 0 \Rightarrow F_y = (M+m)g - T_y$

$= (M+m)g - T\sin\theta$

Plugging in numbers

$\Rightarrow \quad F_x = 432\ N \qquad F_y = 19.2\ N$

Force acting on rod by wall $= \vec{F}$

$\Rightarrow \quad \vec{F}' =$ force acting on wall by rod $= -\vec{F}$