HW 1

Note Title 9/16/2006

① $\boxed{\text{TM } 1-42}$

$$[T] = [G]^a [M_S]^b [r]^c$$

$$s = \left[\frac{kg\ m^3}{kg^2\ s^2}\right]^a kg^b\ m^c$$

$$\begin{aligned} s &: \quad 1 = -2a \Rightarrow \\ kg &: \quad 0 = -a + b \Rightarrow \\ m &: \quad 0 = 3a + c \Rightarrow \end{aligned} \quad \boxed{\begin{aligned} a &= -1/2 \\ b &= -1/2 \\ c &= 3/2 \end{aligned}}$$

② $\boxed{\text{TM } 1-61}$

a) $T = C r^n$

$$0.44\,y = C\,(0.088\text{ Gm})^n$$

$$1.61\,y = C\,(0.208\text{ Gm})^n$$

$$\frac{0.44}{1.61} = \left(\frac{0.088}{0.208}\right)^n$$

$$n = \frac{\ln\left(\frac{0.44}{1.61}\right)}{\ln\left(\frac{0.088}{0.208}\right)} \approx \boxed{1.5}$$

$$C \approx \frac{0.44\,y}{(0.088\,\text{Gm})^{3/2}} \approx \boxed{17\,\frac{y}{(\text{Gm})^{3/2}}}$$

b) $6.20\,y = 17 \dfrac{y}{(Gm)^{3/2}} r^{3/2}$

$$\boxed{r = 0.51\,Gm}$$

3 TM 1-62

a)

$$[T] = [L]^a [g]^b$$

$$s = m^a \left(\frac{m}{s^2}\right)^b$$

$$\begin{aligned} m &: 0 = a + b \\ s &: 1 = -2b \end{aligned} \implies \boxed{\begin{aligned} b &= \tfrac{-1}{2} \\ a &= \tfrac{1}{2} \end{aligned}}$$

b)

T	L
1.9 s	89 cm
1.3 s	44.5 cm

Used thread and key. Took 10 oscill. measurements

as expected

c) $T = c\sqrt{\dfrac{L}{g}}$

$$\boxed{c \approx 2\pi}$$

④ a) $f(x) = 1 - \cos(x)$

$$f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(x_0)}{n!} (x - x_0)^n$$

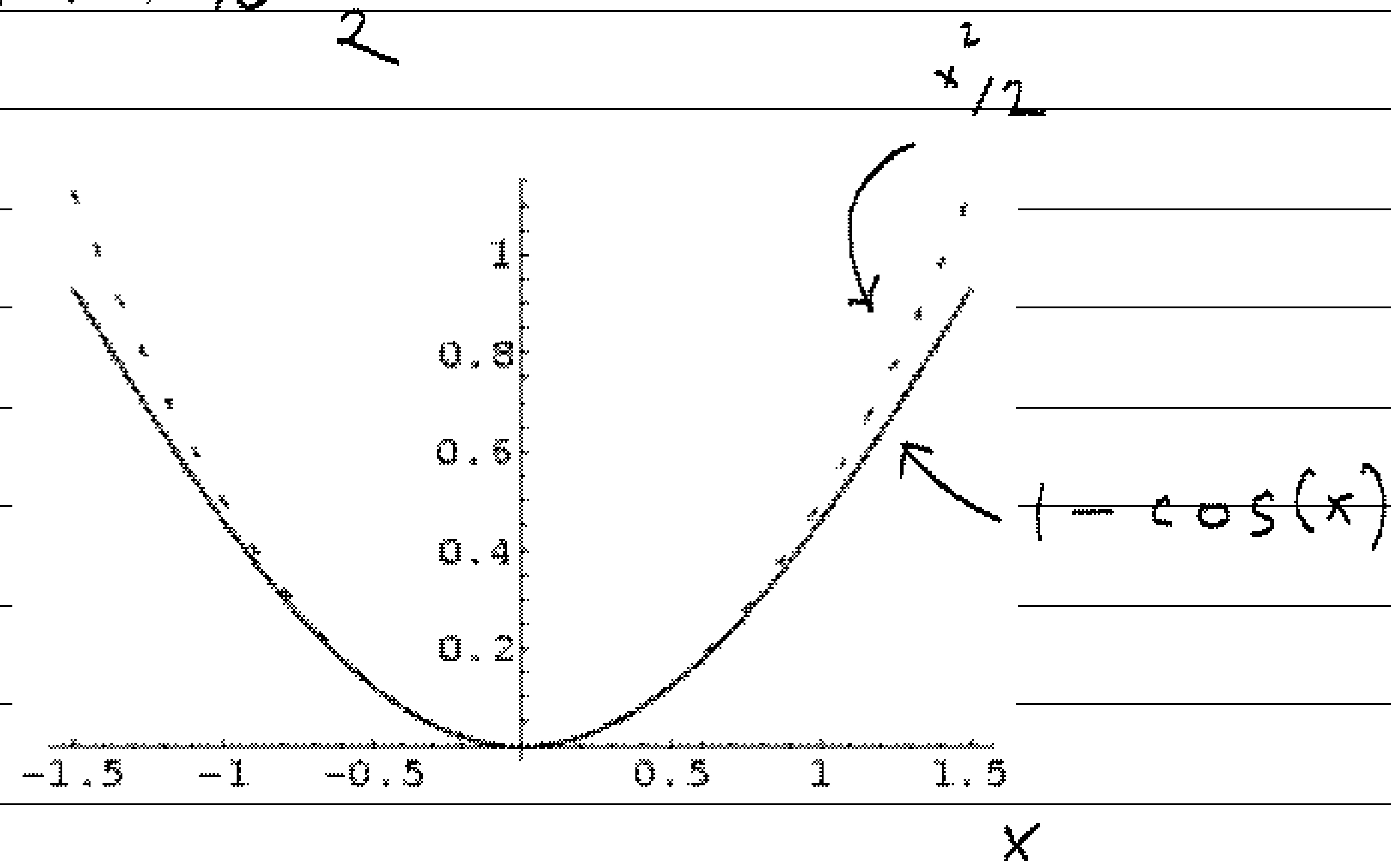
$x_0 = 0$ since expanding about 0

$$f^{(0)}(x_0) = 1 - \cos(x_0) = 0$$
$$f^{(1)}(x_0) = \sin(x_0) = 0$$
$$f^{(2)}(x_0) = \cos(x_0) = 1$$

Hence to second order,

$$f(x) \approx \frac{x^2}{2}$$

b)



⑤ $\boxed{\text{TM } 2-69}$

a)

	$\Delta V\ (\frac{m}{s})$	$\Delta t\ (s)$	$a_{av}\ (\frac{m}{s^2})$
AB	10	3	$10/3$
BC	0	3	0
CE	−30	4	$-15/2$

b) $$\Delta X = \int_0^{10\,s} v\,dt = \text{area under the curve}$$

only need to calculate from A to C

$$\Delta X = \left(5 \times 3 + \frac{10 \times 3}{2} + 15 \times 3\right) \text{ m}$$

$$= \boxed{75 \text{ m}}$$

c) In the interval of A to B, we have

$$v = v_{o_{AB}} + a_{AB}\, t$$

$$\Delta X = \int_0^t dt\, v = v_{o_{AB}} t + \frac{1}{2} a_{AB} t^2$$

$$v_{o_{AB}} = 5 \frac{m}{s} \qquad a_{AB} = 10/3 \ \frac{m}{s^2}$$

In the interval B to C, we have

$$\Delta X = X_B + v_B (t - t_B)$$

where

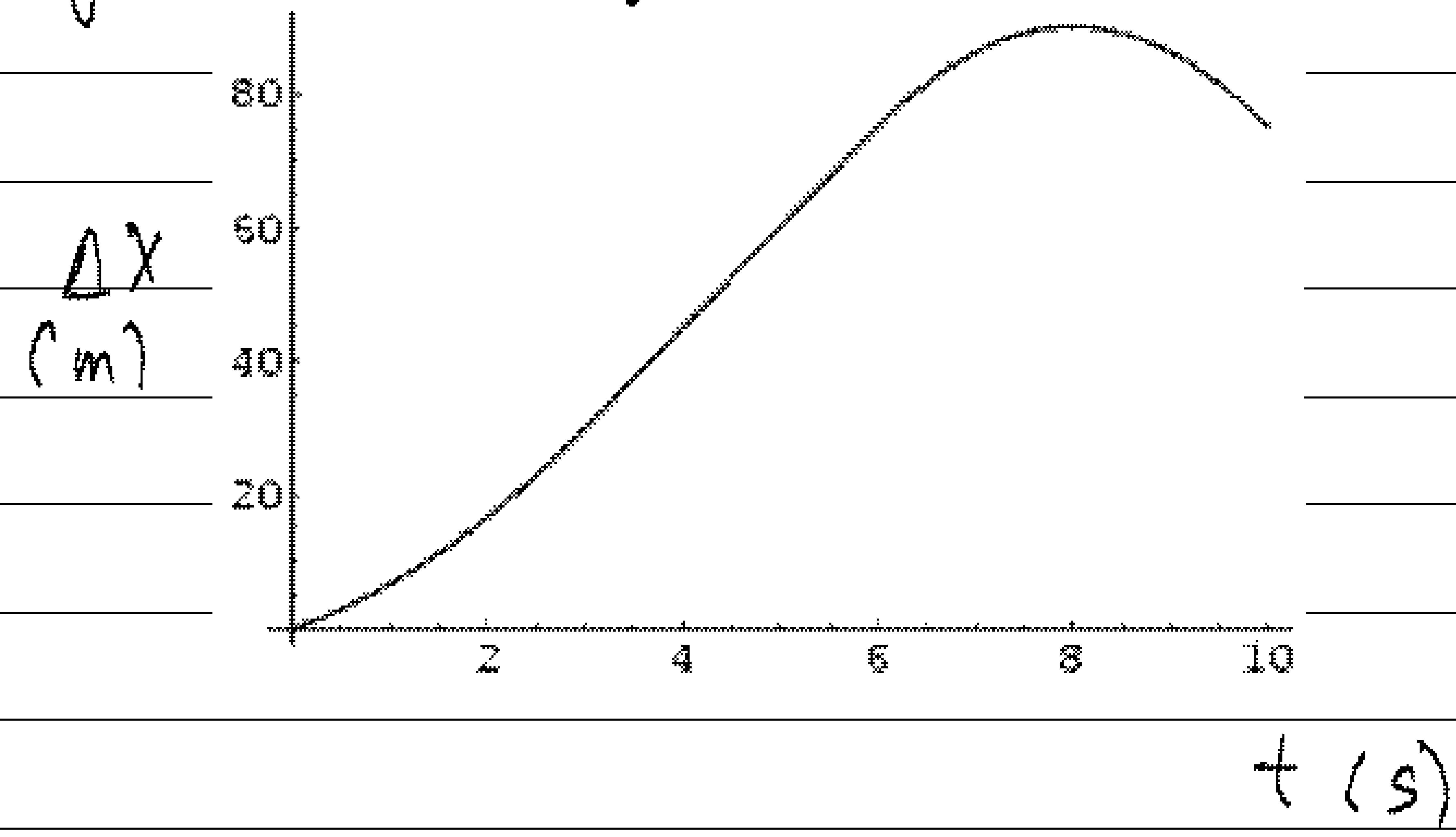
$$t_B = 3\text{ s} \qquad v_B = 15 \frac{m}{s} \qquad X_B = 30 \text{ m}$$

In the interval C to E, we have

$$\Delta x = X_C + v_C (t - t_C) + \frac{1}{2} a_{CE} (t - t_C)^2$$

$$t_C = 6\text{ s} \qquad v_C = 15 \frac{m}{s} \qquad a_{CE} = -15/2 \ \frac{m}{s^2} \qquad X_C = 75 \text{ m}$$

Putting everything together, we find



d) The particle is travelling most slowly at point D.

(6) $\boxed{\text{TM } 2-83}$

a) $$\Delta x = \frac{1}{2} a_1 (\Delta t_1)^2 + v_2 \Delta t_2 + \frac{v_3^2}{2 a_3}$$

$a_1 = 1.5 \frac{m}{s^2}$ $\Delta t_1 = 12 s$

$v_2 = a_1 \Delta t_1 = 18 \frac{m}{s}$ $\Delta t_2 = 25 s$

$a_3 = 1.5 \frac{m}{s^2}$ $v_3 = v_2$

$$\boxed{\Delta x = 666 \text{ m}}$$

b) $$v_{av} = \frac{\Delta x}{\Delta t}$$

$$\Delta t = 12\,s + 25\,s + \frac{v_3}{a_3} = 49\,s$$

$$\therefore \boxed{v_{av} = \frac{666}{49}\ \frac{m}{s}}$$

(7) $\boxed{\text{TM 2-103}}$

a) $v_i = 60\ \frac{m}{s}$ $\Delta x_{max} = 70\ m$

$$\therefore a_{min} = \frac{-v_i^2}{2\Delta x_{max}} = \boxed{\frac{-180}{7}\ \frac{m}{s^2}}$$

b) $$-a_{min}\,\Delta t = v_i$$

$$\boxed{\Delta t = \frac{7}{3}\ s}$$

(8) $\boxed{\text{TM 2-108}}$

$$h_A = H - \frac{1}{2} g (\Delta t)^2 \qquad |v_A| = g\,\Delta t$$

$$h_B = v_0\,\Delta t - \frac{1}{2} g (\Delta t)^2 \qquad |v_B| = v_0 - g\,\Delta t$$

$h_A = h_B$ since colliding

$$H = v_0\,\Delta t$$

$$|v_A| = 2\,|v_B|$$

$$\frac{1}{2} g\,\Delta t = v_0 - g\,\Delta t$$

$$\frac{3}{2} g\,\Delta t = v_0$$

Using eq. for H, $\therefore \quad H = \frac{3}{2} g(\Delta t)^2$

$$\therefore \quad h_{A\,coll} = H - \frac{H}{3} = \boxed{\frac{2}{3} H}$$

(9) $\boxed{\text{TM } 2-130}$

$$\frac{d^2h}{dt^2} = -g e^{-bt}$$

Integrating, $\dfrac{dh}{dt} = -\dfrac{g}{b}\left[1 - e^{-bt}\right]$

where we have used initial velocity being 0 (negative height means sinking).

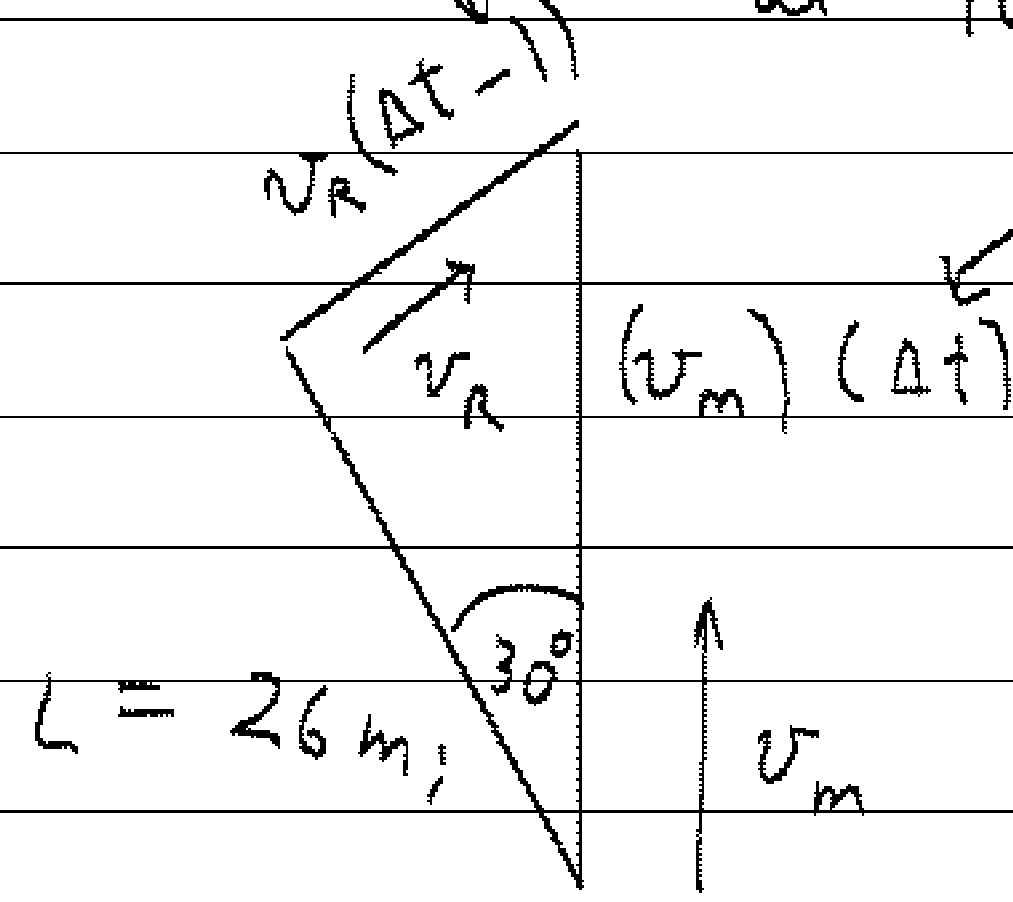
Integrating again,

$$\boxed{h = -\frac{g}{b}\left[t - \frac{1}{b}\left(1 - e^{-bt}\right)\right]}$$

(10) $\boxed{\text{TM } 3-62}$

1 less because left at 10 AM

left at 9 AM



$v_m = 8$ mi/h

$v_R = 6$ mi/h

$$[v_R(\Delta t - 1)]^2 = (L)^2 + (v_m \Delta t)^2 - 2(L)(v_m \Delta t)\cos 30°$$

$$v_R^2(\Delta t^2 - 2(\Delta t)1 + 1) = (L)^2 + v_m^2(\Delta t)^2 - \sqrt{3}L(v_m \Delta t)$$

$$0 = (\Delta t)^2(v_m^2 - v_R^2) + \Delta t(-\sqrt{3}L v_m + 2v_R^2 1) - v_R^2 1^2 + L^2$$
$$= (\Delta t)^2 a + \Delta t\, b + c$$

$$\Delta t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$b = 2v_R^2 1 - \sqrt{3}L v_m = -288 \frac{mi^2}{h}$$

$$a = v_m^2 - v_R^2 = 28 \frac{mi^2}{h^2}$$

$$c = L^2 - v_R^2 1^2 = 640\ mi^2$$

$$\Delta t = 5.14 \pm 1.90\ h$$

To find the correct sign, note that in the limit $v_m \to 0$, we must have

$$\Delta t = 1 + \frac{L}{v_R}$$

since Mary has not gone anywhere.

$$b = 2 v_R^2\, 1 \qquad a = -v_R^2$$

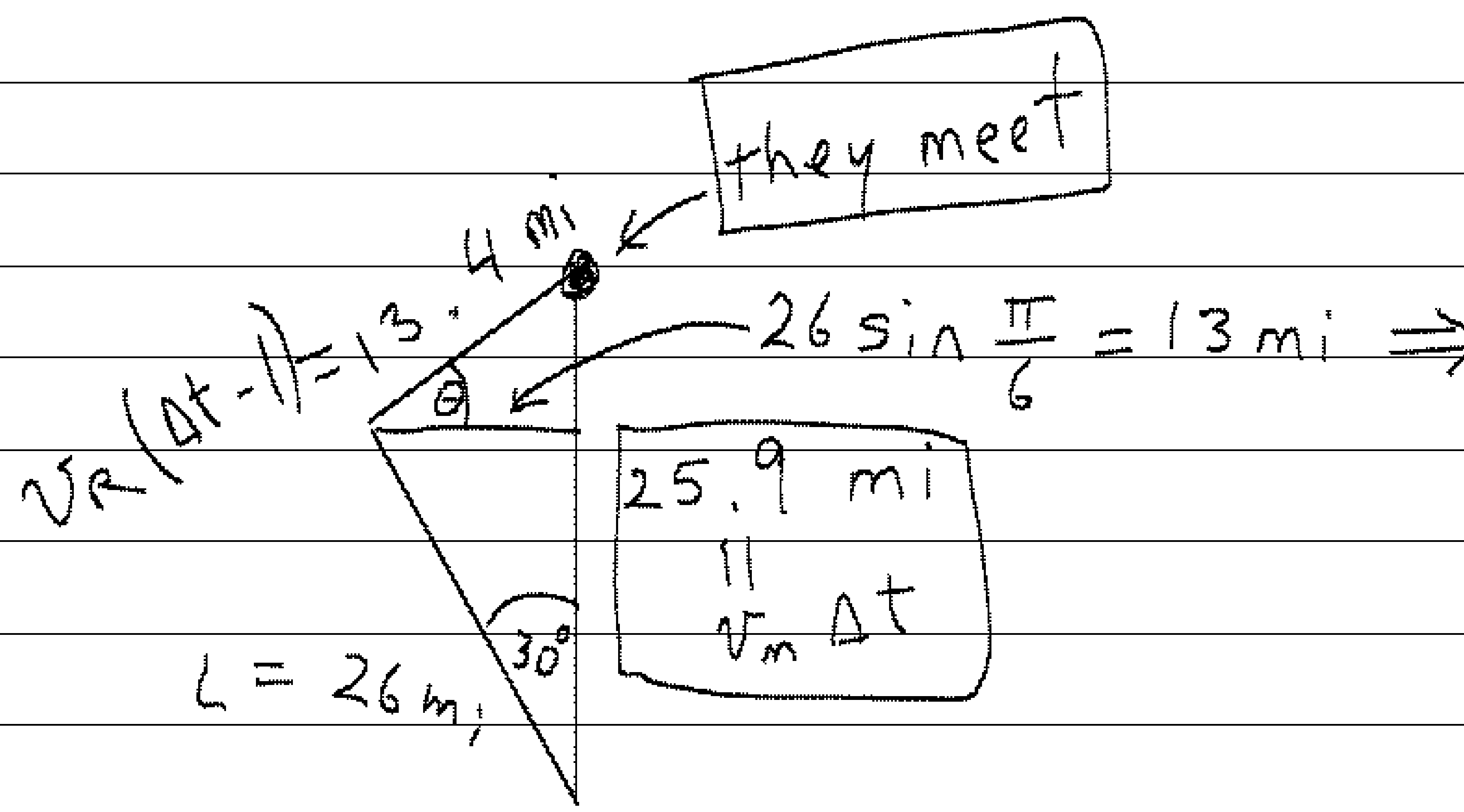
$$\Delta t = \frac{-2 v_R^2\, 1}{-2 v_R^2} \pm \frac{\sqrt{4 v_R^4 + 4 v_R^2 (L^2 - v_R^2 1^2)}}{-2 v_R^2}$$

$$= 1 \pm \frac{\cancel{2} v_R L}{-\cancel{2} v_R^2}$$

Hence, we must take the negative sign.

$$\Delta t = 5.14 - 1.90\text{h} = \boxed{3.24\text{h}}$$

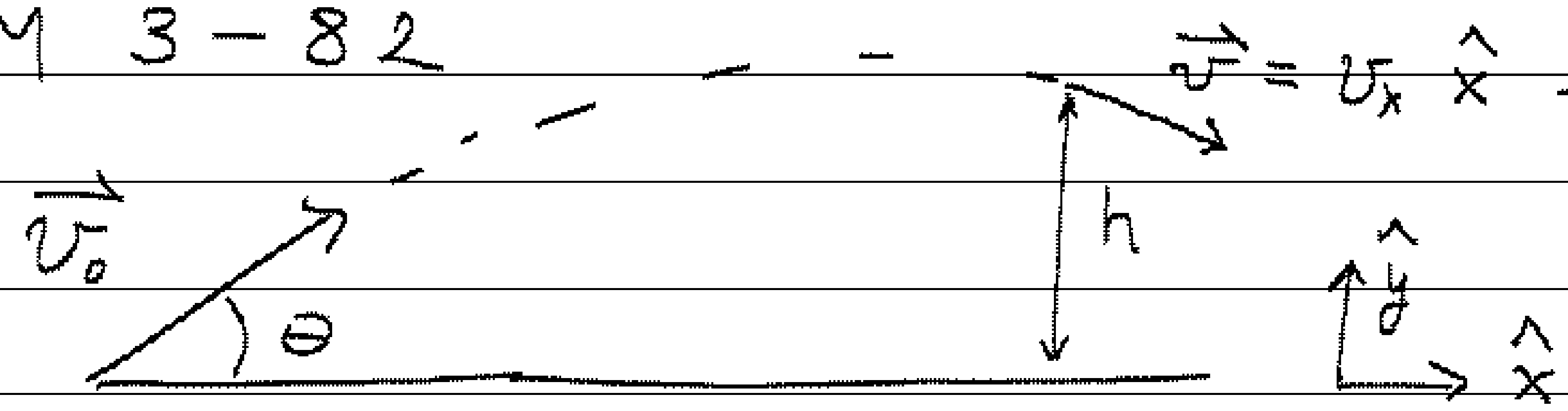
or around $\boxed{12:15 \text{ PM}}$



$\boxed{\theta \approx 14° \text{ N of east}}$

is Robert's direction

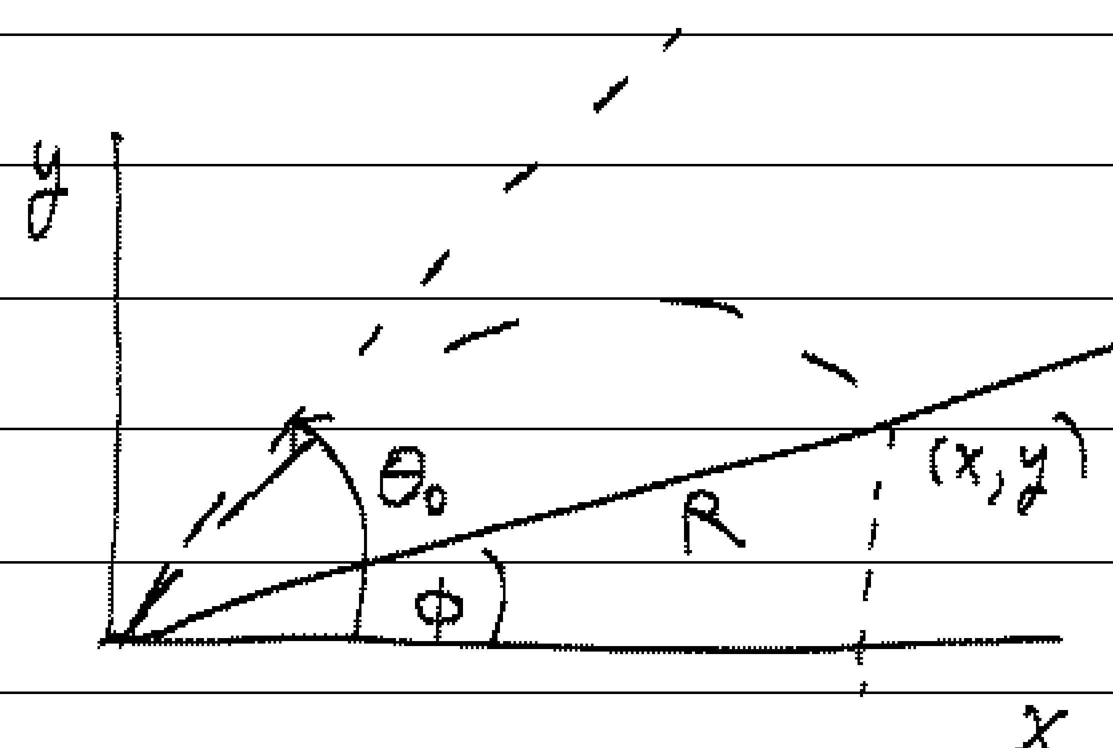
⑪ TM 3-82



$\vec{v} = v_x \hat{x} + v_y \hat{y}$

$$h = (v_0 \sin\theta)\,\Delta t - \frac{1}{2} g (\Delta t)^2$$
$$v_x = v_0 \cos\theta$$
$$v_y = v_0 \sin\theta - g(\Delta t)$$
$$|\vec{v}| = \sqrt{v_0^2 \cos^2\theta + (v_0 \sin\theta - g(\Delta t))^2}$$
$$= \sqrt{v_0^2 + g^2(\Delta t)^2 - 2 g v_0 \sin\theta\,\Delta t}$$
$$= \sqrt{v_0^2 + \left[\frac{1}{2} g (\Delta t)^2 - v_0 \sin\theta\,\Delta t\right] 2g}$$
$$= \boxed{\sqrt{v_0^2 - 2gh} \quad \text{independent of } \theta}$$

⑫ $\boxed{\text{TM } 3-102}$



$$x = v_0 \cos\theta_0\,\Delta t$$
$$y = v_0 \sin\theta_0\,\Delta t - \frac{1}{2} g (\Delta t)^2$$

$$\frac{y}{x} = \tan\phi$$

$$\tan\theta_0 - \frac{\frac{1}{2} g \Delta t}{v_0 \cos\theta_0} = \tan\phi$$

$$R = \sqrt{x^2 + y^2}$$

$$= x\sqrt{1+\tan^2\phi} = x\sec\phi$$

$$= v_0 \cos\theta_0 \Delta t \sec\phi = \boxed{\frac{2v_0^2\cos^2\theta_0}{\cos\phi}\left[\frac{\tan\theta_0 - \tan\phi}{g}\right]}$$

(13) $\vec{A} = (1,2,3)$ $\vec{B} = (2,-2,3)$

a) $$\vec{A}\times\vec{B} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ 1 & 2 & 3 \\ 2 & -2 & 3 \end{vmatrix} = \hat{x}(6+6) + \hat{y}(6-3) + \hat{z}(-2-4)$$

$$= \boxed{12\hat{x} + 3\hat{y} - 6\hat{z}}$$

b) $$|\vec{A}\times\vec{B}| = \sqrt{144+9+36} = \boxed{3\sqrt{21}}$$

c) $$(\vec{A}\times\vec{B})\times\vec{A} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ 12 & 3 & -6 \\ 1 & 2 & 3 \end{vmatrix}$$

$$= (9+12)\hat{x} + (-6-36)\hat{y} + (24-3)\hat{z}$$

$$= \boxed{21\hat{x} - 42\hat{y} + 21\hat{z}}$$

d) $$\vec{A}\cdot\vec{B} = 2-4+9 = \boxed{7}$$