PROBLEM SET 1

due: Monday, Jan 30, 2009, at the beginning of lecture

Problems

(1) Consider the choice of units where $G_N = \hbar = 1$ but not c. Recall that in the SI units, we have

$$[\hbar] = kg\, m^2/s$$
$$[G_N] = m^3/kg/s^2$$

(a) Write down the time dependent nonrelativistic Schroedinger equation (in the spacetime position basis) for a point particle wave function $\psi(t, \vec{x})$ of mass m in the presence of a central gravitational potential due to a point mass $M \gg m$ fixed at the origin.

(b) What is the power n for the units distance^n describing energy?

(c) Suppose you are given that one eigenenergy of the system has the value E_0 in the present units. What numerical value must be multiplied to E_0 to conver the energy value to a Joule?

(2) It is easy to see that

$$\epsilon_{ijk}\partial_j A_k = (\vec{\nabla} \times \vec{A})_i.$$

(a) Prove the identity

$$\epsilon_{ijk}\epsilon_{lmk} = \delta_{il}\delta_{jm} - \delta_{im}\delta_{jl}$$

(b) Use the index notation to show that

$$\vec{\nabla} \times (\vec{\nabla} \times \vec{A}) = \vec{\nabla}(\vec{\nabla} \cdot \vec{A}) - \vec{\nabla}^2 \vec{A}$$

(3) Verify that the proper homogeneous Lorentz boosts

$$\Lambda^i{}_j = \delta_{ij} + v_i v_j \frac{(\gamma - 1)}{\vec{v}^2}$$
$$\Lambda^0{}_j = \gamma v_j$$

satisfy

$$\eta_{\lambda\gamma} = \eta_{\alpha\beta}\Lambda^\alpha{}_\lambda \Lambda^\beta{}_\gamma.$$