PHYSICS 717 PROBLEM SET 10

due: Monday, April 13, 2009, at the beginning of lecture

Problems

(1) Big bang singularity:
  (a) Show that a diffeomorphism invariant scalar made of curvature tensor diverges for the FRW cosmology with a equation of state $w = 1/3$ for a perfect fluid.
  (b) Show that the singularity is spacelike.

(2) Show that the spatial part of the FRW metric for $K = 1$ corresponds to the metric of a 3-sphere embedded in Euclidean 4-space. (i.e. Equation for a 3-sphere in Euclidean 4-space is
$$R^2 = x_1^2 + x_2^2 + x_3^2 + x_4^2$$
where R is the radius of the 3-sphere.)

(3) Because electrons combined with protons to form neutral hydrogen at temperature of about 0.3 eV, majority of the cosmic background photons that we see last scattered off of charged particles at that temperature. This is called the "last scattering surface." (All answers should be accurate at least to an order of magnitude.)
  (a) Given that the temperature of the cosmic background photon today is about 2.3×10^{-4}eV, compute the redshift z_* at the last scattering surface.
  (b) Compute the physical distance (on the homogeneous spacelike hypersurface today) from Earth to the last scattering surface. (i.e. The last scattering surface occupies a fixed coordinate distance with respect to us today. Compute the physical distance between that surface and us where the spatial metric is defined on the homogeneous spacelike hypersurface today.)
  (c) How far can a photon travel from the time of big bang singularity to the time of the last scattering surface? If you call that distance X, a causally connected volume region at the time of last scattering surface is X^3 (assuming causal processes started at the big bang singularity).
  (d) About how many causally disconnected patches are there at the last scattering surface. [Use results of part c).]

(4) Find a solution to the Einstein's equations if the stress tensor is dominated by a cosmological constant. Write the solution in the form
$$ds^2 = -dt^2 + g_{ij} dx^i dx^j$$
and find g_{ij}.

(5) Show that a self-gravitating over density relative to an average background density $\langle \rho \rangle$, i.e.
$$\delta(x) \equiv \frac{\rho(x) - \langle \rho \rangle}{\langle \rho \rangle}$$
will grow linearly with the expansion of a matter dominated universe.