# PHYSICS 717 PROBLEM SET 11

**due:** Monday, April 20, 2009, at the beginning of lecture

Problems

**1.:** Derive starting with

$$G_{ab} = 8\pi T_{ab}$$

and

$$g_{\alpha\beta} = \eta_{\alpha\beta} + h_{\alpha\beta}$$

the linearized equation

$$-\frac{1}{2}\partial_\lambda \partial^\lambda \bar{h}_{\alpha\beta} + \partial^\lambda \partial_{(\beta}\bar{h}_{\alpha)\lambda} - \frac{1}{2}\eta_{\alpha\beta}\partial^\lambda \partial^\gamma \bar{h}_{\lambda\gamma} = 8\pi T_{\alpha\beta}$$

where

$$\bar{h}_{\alpha\beta} \equiv h_{\alpha\beta} - \frac{1}{2}\eta_{\alpha\beta} h.$$

**(Hint::** Evaluate the Ricci tensor and Ricci scalar in the frame in which $\Gamma^\lambda_{\alpha\beta} = 0$. )

**2.:** Recall from lecture that the Lorentz gauge residual gauge fixing conditions $h = 0$ and $h_{\mu 0} = 0$ can both be satisfied simultaneously. Check this explicitly by showing that

$$\left\{ A_{00} + 2i\beta_0 k_0 = \frac{-1}{2} A^\alpha{}_\alpha,\ i\left[\beta_i k_0 + \beta_0 k_i\right] = -A_{0i} \right\}$$

(which comes from $h_{0\alpha} = 0$) together with $\partial_\mu \bar{h}^{\mu\nu}$=0 implies

$$i\beta^\gamma k_\gamma = \frac{1}{2} A^\alpha{}_\alpha$$

which is equivalent to $h = 0$. (The definition of the notation is from lecture.)

**3.:** Green's function exercise: Suppose you are told that the gravitational waves in a world different from ours obey the following equation:

$$\partial^\lambda \partial_\lambda \bar{h}_{\alpha\beta}(t, \vec{x}) - m^2 \bar{h}_{\alpha\beta}(t, \vec{x}) = -16\pi T_{\alpha\beta}$$

with an external source given by

$$T_{\alpha\beta} = M\Theta(t)\delta^{(3)}(x)\delta_{\alpha 0}\delta_{\beta 0}$$

where {M, $m$} are positive constants. ($\Theta(t)$ is a step function which is unity when $t > 0$ and zero otherwise). Compute the time average

$$\langle \bar{h}_{\alpha\beta} \rangle \equiv \lim_{T\to\infty} \frac{1}{T}\int_0^T dt \bar{h}_{\alpha\beta}(t, \vec{x})$$

for the solution satisfying boundary conditions $\bar{h}_{\alpha\beta}|_{t=-1/m} = 0$ and $\partial_t \bar{h}_{\alpha\beta}|_{t=-1/m} = 0$. You may find the following integral useful:

$$\int_{-\infty}^{\infty} dy \frac{y \sin(yc)}{1 + y^2} = \pi e^{-c}$$

for $c > 0$.

**4.:** Calculate the gravitational radiation luminosity of a spining thin uniform metal rod of mass $M$ and length $l$, spinning at frequency $w$ around a symmetrical perpendicular axis. Estimate the electromagnetic luminosity which would arise from the slight excess of electrons pushed toward the ends by "centrifugal force." (Note the electromagnetic radiation is approximately quadrupole in this case since the dipole charge distribution is approximately zero.) If the rod has a reasonable density (10g/cm$^3$) and is rotating at a reasonable frequency (1 kHz), will electromagnetic or gravitational radiation be more important in slowing the rotation?