PROBLEM SET 2

due: Monday, Feb 9, 2009, at the beginning of lecture

Problems

(1) Lesson in signs: Show that the $\Lambda^{\mu}{}_{\nu}(\vec{v})$ in problem 3 of the last homework has the interpretation of boosting particle velocities (making objects at rest move at speed $\vec{v}$). Such boosts are sometimes referred to as active boost transformations, as the substitution

$$dx^{\mu} \rightarrow dx'^{\mu} = \Lambda^{\mu}{}_{\nu} dx^{\nu}$$

changes the state of the system. Show that from a coordinate change perspective, $\Lambda^{\mu}{}_{\nu}(\vec{v})$ defined according to

$$dx'^{\mu} = \Lambda^{\mu}{}_{\nu}(\vec{v}) dx^{\nu} = \begin{pmatrix} \gamma & -\gamma\vec{v} \\ -\gamma\vec{v} & \delta^{ij} + v^i v^j \frac{\gamma-1}{|\vec{v}|^2} \end{pmatrix} \begin{pmatrix} dt \\ d\vec{x} \end{pmatrix} \tag{0.1}$$

has the interpretation of passive transformation, in which there is simply a coordinate transformation between old coordinates and new coordinates in which the new coordinate origin is moving at velocity $\vec{v}$ with respect to the old coordinate origin. For example, what is the 3-velocity of a particle at rest transformed according to Eq. (0.1)?

(2) If two frames move with 3-velocities $\vec{u}$ and $\vec{w}$, show that their relative velocity magnitude is given by

$$\vec{v}^2 = \frac{(\vec{u}-\vec{w})^2 - (\vec{u}\times\vec{w})^2}{(1-\vec{u}\cdot\vec{w})^2}$$

(3) What is the threshold kinetic energy K_{th} of the incident electron for the following process to occur? electron (fast) + proton (at rest) $\rightarrow$ electron + antiproton + two protons? Express your answer in terms of the masses of proton (= mass of antiproton) and electron.

(4) Exercises with the Field strength tensor.

(a) Show that for an observer moving with a 4-velocity u^a, the quantity $E_a \equiv F_{ab}u^b$ is interpreted as the electric field and $B_a = \frac{-1}{2}\epsilon_{ab}{}^{cd}F_{cd}u^b$ is interpreted as the magnetic field.

(b) Suppose a given $F^{\mu\nu}$ in frame O is observed in a frame $\bar{O}$ which is moving with a velocity $v\hat{x}$ with respect to O. Evaluate explicitly the matrix components of $\bar{F}^{\mu\nu}$ in terms of E^i, B^i, v, and γ.

(c) Show that

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

(5) Show that

$$f^{\alpha} = eF^{\alpha}{}_{\gamma}\frac{dx^{\gamma}}{d\tau}$$

and

$$\frac{dp^{\alpha}}{d\tau} = f^{\alpha}$$

expresses the Lorentz force law

$$\frac{d\vec{p}}{dt} = e[\vec{E} + \vec{v}\times\vec{B}].$$

(6) Suppose person B is moving at constant speed v along the x-axis with respect to person A. Person A throws a ball to person B, and person B throws the ball back immediately. Suppose the speed of the ball (call it u) exceed the speed of light in the thrower's rest frame.

(a) Calculate the roundtrip time assuming that person B is a distance L away when he throws the ball back.

(b) Can the roundtrip time be negative?

(7) Recall that Minkowski space is a spacetime where the geometry (Lorentz invariant distances) is given by

$$ds^2 = \eta_{\alpha\beta}dx^{\alpha}dx^{\beta}$$

where the components of $\eta_{\alpha\beta}$ is given by

$$\eta_{\alpha\beta} = \begin{pmatrix} -1 & & & \\ & 1 & & \\ & & 1 & \\ & & & 1 \end{pmatrix}.$$

Any time we write a length in the form

$$ds^2 = g_{\mu\nu}dx^{\mu}dx^{\nu}$$

where dx^{μ} correspond to an infinitesimal coordinate differential, we call $g_{\mu\nu}$ a metric. Suppose we use the coordinates (u, v, y, z) to cover Minkowski space instead of (t, x, y, z) where $u = t - x$ and $v = t + x$. What are the components of the metric in this space?