# Physics 717 Problem set 3

February 9, 2009

**due** Monday, Feb 16, 2009, at the beginning of lecture

Problems

1. Using the definition
$$g_{\mu\nu} = \frac{\partial \xi^\alpha}{\partial x^\mu}\frac{\partial \xi^\beta}{\partial x^\nu}\eta_{\alpha\beta},$$
show that
$$g_{\mu\nu;\lambda} = 0.$$
2. Show by explicit construction that two coordinate systems suffice to cover the two-sphere $S^2$(surface of a ball in Euclidean space).
3. Is
$$\bar{x}(x,t) = x - \frac{1}{2}gt^2$$
$$\bar{t}(x,t) = t$$
an example of a diffeomorphism? Why or why not?
4. Problem 3 b),c) on page 27 of Wald.
5. (Messy problem !)Properties of the affine connection:
   (a) Compute $\Gamma^\mu_{\alpha\beta}$ for the Minkowski metric
$$g_{\mu\nu} = \begin{pmatrix} -1 & & & \\ & 1 & & \\ & & 1 & \\ & & & 1 \end{pmatrix}$$
   (b) What is $g_{\mu\nu}$ for the Minkowski metric if the coordinates are changed from that of part (a) to
$$t' = t$$
$$x' = \sqrt{x^2+y^2}\cos(\phi - wt)$$
$$y' = \sqrt{x^2+y^2}\sin(\phi - wt)$$
$$z' = z$$
   where $\tan\phi = y/x$
   (c) Compute $\Gamma^\mu_{\alpha\beta}$ in the coordinates of part (b).
   (d) Based on the answers to different parts of this problem, can you tell whether $\Gamma^\mu_{\alpha\beta}$ is a tensor?