

Physics 717 Problem set 4

February 13, 2009

due Monday, Feb 23, 2009, at the beginning of lecture

1. Wald Chapter 3, problem 1.
2. On the surface of a 2-sphere, the Euclidean metric is given by

$$ds^2 = d\theta^2 + \sin^2 \theta d\phi^2$$

Suppose a vector $\vec{A} = \hat{e}_\theta$ at $(\theta = \theta_0, \phi = 0)$. What is \vec{A} after it is parallel transported around the circle at $\theta = \theta_0$? What is its magnitude?

3. (tedious problem) Compute the Riemann tensor component

$$R^0_{101}$$

if the metric is given as

$$ds^2 = a^2(\eta)(-d\eta^2 + d\vec{x}^2).$$

We will see later that this is the average metric for the universe.

4. Show that

$$R_{abcd} = R_{cdab}$$

5. Consider another common definition of the Riemann tensor $R : V \otimes V \otimes V \rightarrow V$ as

$$R(X, Y, Z) \equiv \nabla_X \nabla_Y Z - \nabla_Y \nabla_X Z - \nabla_{[X, Y]} Z$$

where $\nabla_X \equiv X^a \nabla_a$ and $X^a \in V$. Show that this is consistent with the Riemann tensor definition of lecture 9 if there is no torsion.

6. In Newtonian gravity, two nearby particles with trajectories $x^i(t)$ and $\bar{x}^i(t)$ in Euclidean coordinates evolve as

$$\frac{d^2 \zeta^i}{dt^2} = - \sum_{j=1}^3 \frac{\partial^2 \phi}{\partial x^i \partial x^j} \zeta^j$$

where Φ is the Newtonian gravitational potential. Use geodesic deviation equation to derive this equation in the weak-field, Newtonian limit of GR, where we have seen that

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

can be used with $|h_{\mu\nu}| \ll 1$ and all particle 3-velocity magnitudes can be considered small.