PHYSICS 717 PROBLEM SET 6

due: Monday, March 9, 2009, at the beginning of lecture

Problems

1.: Compute the connection 1-forms of the metric

$$ds^2 = -e^{2\Lambda(t,r)}dt^2 + e^{2\Phi(t,r)}dr^2 + e^{2\Gamma(t,r)}(d\theta^2 + \sin^2\theta d\phi^2).$$

2.: Show that Bianchi identity $\nabla_{[a}R_{bc]d}{}^{e} = 0$ follows from considering

$$dR_{\mu}{}^{\nu}$$

where $R_{\mu}{}^{\nu}$ is a curvature 2-form defined in lecture 16.

3.: Express $G^0{}_2$ and $G^2{}_2$ in terms of Riemann tensor components. E.g. in class we found

$$G^0{}_0 = -(R^{12}_{12} + R^{13}_{13} + R^{23}_{23}).$$

4.: Compute the $G^0{}_0$ Einstein equation resulting from

$$ds^2 = -dt^2 + a^2(d\chi^2 + \sin^2\chi[d\theta^2 + \sin^2\theta d\phi^2]),$$

using the results of lectures 16 and 17.

5.: Consider a 2-sphere of unit radius with the natural metric

$$ds^2 = d\theta^2 + \sin^2\theta d\phi^2.$$

Find 3 linearly independent Killing vectors.

6.: Show that any Killing vector ξ^ν is a solution of the equation

$$\nabla^\mu\nabla_\mu\xi^\nu + R^\nu{}_\sigma\xi^\sigma = 0.$$

Compare and contrast with the equation for the vector potential of electrodynamics.