PHYSICS 717 PROBLEM SET 12

due: Friday, May 1, 2009, at the beginning of lecture

Problems

1.: Compute the stress tensor for the weak field gravity waves in vacuum starting from the action derived in class:

$$S = \frac{M_p^2}{32\pi} \int d^4x \left\{ \frac{1}{2} \partial_\lambda h^{\mu\nu} \partial^\lambda h_{\mu\nu} - \frac{1}{2} \partial_\lambda h \partial^\lambda h - \partial_\lambda h^{\lambda\nu} \partial^\mu h_{\mu\nu} + \partial^\nu h \partial^\mu h_{\mu\nu} \right\}$$

in radiation gauge. (hint: Allow $\eta_{\mu\nu}$, the metric of the background on which $h_{\mu\nu}$ is propagating, to vary away from the flat metric and compute

$$T_{\mu\nu} = \frac{2}{\sqrt{g}} \frac{\delta S}{\delta g^{\mu\nu}}$$

and evaluate the functional with $h_{\mu\nu}$ set to vacuum radiation gauge solution.)

2.: Using Noether's procedure with the $SO(3,1)$ Lie group parameterization matrix ω (in the notation of lecture 36) given by

$$\omega^{\mu\alpha} = \epsilon(\delta^{\ \mu}_{i} \delta^{\alpha}_{\ j} - \delta^{\ \alpha}_{i} \delta^{\mu}_{\ j}),$$

find the angular momentum density (angular momentum per unit spatial volume) for gravity waves in radiation gauge expressed in terms of $h_{\mu\nu}$. Use the answer to problem 1 to identify the orbital angular momentum part and the spin angular momentum part.

3.: For the following plane wave solution in radiation gauge

$$h_{\mu\nu} = A \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & \cos(k_z(t-z)) & \sin(k_z(t-z)) & 0 \\ 0 & \sin(k_z(t-z)) & -\cos(k_z(t-z)) & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

compute $J^0_{(12)}$ and T^{0z}. Interpret the result, given that from a quantum perspective $T^{0z}/(\hbar k_z)$ counts the number of gravitons per unit volume.

4.: Explicitly check that

$$[\Sigma^{\alpha\beta}, \Sigma^{\gamma\delta}] = \eta^{\gamma\beta}\Sigma^{\alpha\delta} - \eta^{\gamma\alpha}\Sigma^{\beta\delta} + \eta^{\delta\beta}\Sigma^{\gamma\alpha} - \eta^{\delta\alpha}\Sigma^{\gamma\beta}$$

where

$$\Sigma^{\alpha\beta} = \frac{1}{4}\left[\gamma^\alpha, \gamma^\beta\right]$$

with the metric signature $(1,-1,-1,-1)$ and γ^α are those given in class.

5.: Suppose the action governing an electron field is

$$S = \int d^4x \sqrt{g} \bar{\psi} \left[i\gamma^\nu D_\nu - m \right] \psi$$

where D_ν is the covariant derivative defined in class. Compute the stress energy tensor for the electron field by varying the tetrad. (Work with the metric signature (1,-1,-1,-1).)