PHYSICS 717 PROBLEM SET 8

due: Monday, March 30, 2009, at the beginning of lecture

Problems

1.: Problem 4 on page 158 of Wald.

2.: Simple coordinate exercises:

a): Show that Schwarzschild curvature singularity at $r = 0$ is a spacelike hypersurface.

b): Show that for Schwarzschild $r < 2M$, surfaces of constant Schwarzschild t are straight lines in Kruskal coordinates and that surfaces of constant r are hyperbolas in Kruskal coordinates.

3.: Referring to the construction of a Kruskal wormhole in Euclidean coordinate embedding, suppose you have the solutions $\{r(t), z(t)\}$ to the equations (where $z(t)$ represents all branches if it involves square roots)

$$(\frac{dr}{dt})^2 + (\frac{dz}{dt})^2 = \frac{2M}{r} - 1$$

$$\sqrt{1 - \frac{r}{2M}} e^{r/(4M)} \cosh\left[t/(4M)\right] = T_1$$

inside the horizon. Find the functions (of t and ϕ) in the Euclidean vector

$$(x(t,\phi), y(t,\phi), z(t,\phi))$$

that can be used to plot the wormhole. [i.e. express the answer including $r(t)$ and $z(t)$]

4.: Write a single algebraic equation (not differential) determining the minimum Euclidean radius of the Krukal wormhole throat constructed in class as a function of T_1 denoted in lecture.

5.: A particle of mass $m > 0$ falls radially towards the horizon of a Schwarzschild black hole of mass M. The geodesic it follows has $E = 0.95$.

a): Find the proper time required to reach $r = 2M$ from $r = 3M$.

b): Find the proper time required to reach $r = 0$ from $r = 2M$.

c): In Schwarzschild coordinate basis, find its 4-velocity components at $r =$ 2.001M.

6.: In Schwarzschild geometry, compute the $\hat{\phi}$ direction tidal acceleration on a pair of closeby $\hat{\phi}$separated particles that are in geodesic motion, if they are released from rest at a coordinate location $(r, \theta = \pi/2, \phi)$? (i.e. Use geodesic deviation.) Suppose the tolerance of human body is a tidal acceleartion of 500 m/s^2. How massive must the black hole be for human beings to survive the $\hat{\phi}$ tidal acc?