Physics 717 Problem set 9

March 31, 2009

due Monday, April 6, 2009, at the beginning of lecture

Problems

1. Complete the argument in lecture in showing that

$$\frac{dP}{dr} = -(\rho + P)\left\{\frac{\frac{4\pi}{3}r^3\rho + 4\pi r^3 P}{r^2\left[1 - \frac{2m(r)}{r}\right]}\right\}$$

   with $m(r)$ given as in lecture for the the equation of state of ρ being a constant independent of P can be integrated to give

$$\frac{\rho + 3P}{\rho + P} = \frac{\sqrt{1 - \frac{2m(r)}{r}}}{\sqrt{1 - \frac{2M}{R}}}.$$

   Be sure to use the boundary conditions discussed in class.

2. From the result of problem 2, we have

$$P(r) = \frac{\rho\left[\sqrt{1 - \frac{2Mr^2}{R^3}} - \sqrt{1 - \frac{2M}{R}}\right]}{3\sqrt{1 - \frac{2M}{R}} - \sqrt{1 - \frac{2Mr^2}{R^3}}}$$

   Integrate the $\Phi'(r)$ disucssed in class to obtain

$$\exp\left[2\Phi(r)\right] = (1 - \frac{2M}{R})\left[\frac{\rho}{\rho + P(r)}\right]^2$$

   using the boundary conditions discussed in class.

3. Verify the statement about spherically symmetric pressureless dust with the boundary conditions given in class collapsing in finite proper time to reach a singularity in finite time: i.e. Integrate

$$(\frac{\dot{a}}{a})^2 + \frac{k}{a^2} = \frac{8\pi}{3}\rho_0(\frac{a_0}{a})^3.$$

   In what sense is $a = 0$ singular?

4. If a photon is emittted from the surface of a spherical star of uniform density ρ, show that the observed redshift must obey the constraint $z < 2$.