Physics 721 Homework 1
Assigned: Wed. 1/19/05
Due: Mon. 1/24/05

1. Suppose you obtain *working in Gaussian units* a solution

$$Z = ck^4|\vec{j}|^2$$

where $|\vec{j}|$ is a physical quantity which has units Asm *in SI,* k is a wave vector (units m^{-1} *in SI*), and Z has units kg m^2/s^3 in SI. Find an expression for Z in SI units.

answer

After setting $c = 1$, the first step is to divide all sources by $\sqrt{4\pi}$. Since Coulomb unit appears in Ampere through $|\vec{j}|$, we see that the only source in the system is $|\vec{j}|$. After dividing $|\vec{j}|$ by $\sqrt{4\pi}$, we need to multiply by appropriate powers of ϵ_0 and μ_0. By dimensional analysis, we write

$$\begin{aligned} \frac{\text{kg}m^2}{s^3} &= [k^4|\vec{j}|^2\mu_0^{n_1}\epsilon_0^{n_2}] \\ &= \frac{1}{m^4}(\frac{C}{s})^2 s^2 m^2 \frac{kg^{n_1}m^{n_1}}{C^{2n_1}}\frac{C^{2n_2}s^{2n_2}}{kg^{n_2}m^{3n_2}} \end{aligned}$$

Hence, the equations for n_1 and n_2 are

$$m:\ n_1 - 4 + 2 - 3n_2 = 2$$

$$s:\ 2n_2 = -3$$

$$C:\ 2 - 2n_1 + 2n_2 = 0$$

$$\text{kg: } n_1 - n_2 = 1$$

The s equation gives

$$n_2 = -3/2$$

and kg equation gives

$$n_1 = -1/2.$$

Hence, the SI expression is

$$\boxed{Z = \frac{1}{4\pi\sqrt{\mu_0\epsilon_0^3}}k^4|\vec{j}|^2}$$

One can introduce $c = 1/\sqrt{\mu_0\epsilon_0}$ if one wishes to write

$$\boxed{Z = \frac{c}{4\pi\epsilon_0}k^4|\vec{j}|^2}$$

2. Jackson 11-3

answer

Two successive Lorentz transformations should give a another Lorentz transformation.

$$\begin{pmatrix} \gamma_1 & -v_1\gamma_1 \\ -v_1\gamma_1 & \gamma_1 \end{pmatrix}\begin{pmatrix} \gamma_2 & -v_2\gamma_2 \\ -v_2\gamma_2 & \gamma_2 \end{pmatrix} = \begin{pmatrix} \gamma & -v\gamma \\ -v\gamma & \gamma \end{pmatrix}$$

$$\begin{pmatrix} \gamma_1\gamma_2(1+v_1v_2) & -\gamma_1\gamma_2(v_1+v_2) \\ -\gamma_1\gamma_2(v_1+v_2) & \gamma_1\gamma_2(1+v_1v_2) \end{pmatrix} = \begin{pmatrix} \gamma & -v\gamma \\ -v\gamma & \gamma \end{pmatrix}$$

Taking the ratio of the 01 an 11 matrix entry, we find

$$\boxed{v = \frac{v_1+v_2}{1+v_1v_2}}$$

which was the expression that we were looking for.