Physics 721 Homework 7
Assigned: Wednesday 3/2/05 (in class)
Due: Mon. 3/7/05

1. Jackson 3.2

2. Jackson 3.9

Below: FUN (ONLY FOR YOUR INFORMATION; NOT PART OF HOMEWORK)

Using the formula that you learned in lecture for Laplacian operator in arbitrary coordinates, write Laplace's equation in the following coordinate system:

$$x = c \sinh\alpha \sin\beta \cos\phi$$

$$y = c \sinh\alpha \sin\beta \sin\phi$$

$$z = c \cosh\alpha \cos\beta$$

where $0 \le \alpha < \infty$, $0 < \beta \le \pi$, and $-\pi < \phi \le \pi$ where c is a constant.

a)

Show that the length element (Pythagorean theorem in differential element) is given by

$$dl^2 = c^2(\sinh^2\alpha + \sin^2\beta)(d\alpha^2 + d\beta^2) + c^2 \sinh^2\alpha \sin^2\beta d\phi^2.$$

Writing this as

$$dl^2 = g_{ij} dx^i dx^j,$$

where $(x^1, x^2, x^3) = (\alpha, \beta, \phi)$ we can write

$$g_{ij} = c^2 \begin{pmatrix} \sinh^2\alpha + \sin^2\beta & & \\ & \sinh^2\alpha + \sin^2\beta & \\ & & c^2 \sinh^2\alpha \sin^2\beta \end{pmatrix}.$$

b) Using

$$\nabla^2\Phi = \sum_{ij} \frac{1}{\sqrt{g}} \partial_i [\sqrt{g} g^{ij} \partial_j] \Phi = 0$$

where

$$g \equiv \det[g_{ij}]$$

$$\sum_k g^{jk} g_{kl} = \delta^j_{\ l}$$

show that the Laplace equation in this coordinate system is

$$\frac{1}{c^2(\sinh^2\alpha + \sin^2\beta)} \{ \frac{1}{\sinh\alpha} \frac{\partial}{\partial\alpha} [\sinh\alpha \frac{\partial\Phi}{\partial\alpha}] + \frac{1}{\sin\beta} \frac{\partial}{\partial\beta} (\sin\beta \frac{\partial\Phi}{\partial\beta}) + (\frac{1}{\sinh^2\alpha} + \frac{1}{\sin^2\beta}) \frac{\partial^2\Phi}{\partial\phi^2} \} = 0.$$