

Homework 2

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$$\begin{aligned}
 \Delta x'^0 \Delta x'^1 &= \left[\gamma (\Delta x^0 - \vec{v} \cdot \Delta \vec{x}) \right] - \\
 &\quad \left[\Delta \vec{x} + \frac{(\gamma-1)}{v^2} (\vec{v} \cdot \Delta \vec{x}) \vec{v} - \gamma \vec{v} \Delta x^0 \right]^2 \\
 &= \gamma^2 (\Delta x^0)^2 + (\vec{v} \cdot \Delta \vec{x})^2 - 2 \Delta x^0 \vec{v} \cdot \Delta \vec{x} \\
 &\quad - \left[(\Delta \vec{x})^2 + \gamma^2 v^2 (\Delta x^0)^2 + \frac{(\gamma-1)^2}{v^4} (\vec{v} \cdot \Delta \vec{x})^2 v^2 \right. \\
 &\quad \left. + 2 \frac{(\gamma-1)}{v^2} (\vec{v} \cdot \Delta \vec{x}) (\vec{v} \cdot \Delta \vec{x}) + -2 \gamma (\vec{v} \cdot \Delta \vec{x}) \Delta x^0 - 2 \frac{(\gamma-1)}{v^2} (\vec{v} \cdot \Delta \vec{x}) \gamma v^2 \Delta x^0 \right] \\
 &= \gamma^2 \left[(\Delta x^0)^2 (1 - v^2) \right] + (\vec{v} \cdot \Delta \vec{x})^2 \left[\gamma^2 - 2 \frac{(\gamma-1)}{v^2} + -\frac{(\gamma-1)^2}{v^4} v^2 \right] \\
 &\quad + (\vec{v} \cdot \Delta \vec{x}) \Delta x^0 \left[-2\gamma^2 + -2\gamma + 2(\gamma-1)\gamma \right] \\
 &\quad + -(\Delta \vec{x})^2 \\
 &= \boxed{(\Delta x^0)^2 - (\Delta \vec{x})^2}
 \end{aligned}$$

2 (Jackson 11.5) $\vec{a} \equiv \frac{d^2 \vec{x}}{dt^2} = \frac{d}{dt} \frac{d\vec{x}}{dt} = \frac{d}{dt} \vec{u}$

$$a_{||} = \frac{d u_{||}}{dt} = \frac{d}{dt} \left[\frac{u'_{||} + v}{1 + \vec{v} \cdot \vec{u}'} \right] \quad \text{where we used eq. (11.31).}$$

Using $dt = \gamma (dt' + \vec{v} \cdot d\vec{x}')$ (eq. 11.19)
 $= \gamma dt' (1 + \vec{v} \cdot \vec{u}')$

$$\begin{aligned}
 a_{||} &= \frac{1}{\gamma (1 + \vec{v} \cdot \vec{u}')} \left[\frac{\frac{d}{dt'} u'_{||} + 0}{1 + \vec{v} \cdot \vec{u}'} + \frac{-(u'_{||} + v) \vec{v} \cdot \frac{d}{dt'} \vec{u}'}{(1 + \vec{v} \cdot \vec{u}')^2} \right] \\
 &= \frac{1}{\gamma (1 + \vec{v} \cdot \vec{u}')} \left[\frac{a'_{||} (1 + \vec{v} \cdot \vec{u}') - (u'_{||} + v) \vec{v} \cdot \vec{a}'}{(1 + \vec{v} \cdot \vec{u}')^2} \right] \\
 &= \frac{1}{\gamma (1 + \vec{v} \cdot \vec{u}')^3} \left[\frac{a'_{||} + a'_{||} \vec{v} \cdot \vec{u}' - \cancel{a'_{||} \vec{v}} - a'_{||} v^2}{\cancel{(1 + \vec{v} \cdot \vec{u}')^2}} \right]
 \end{aligned}$$