Official Solutions

$\boxed{\text{HW 4}}$

$\boxed{1}$ Current in an ~~fin~~ infinite wire

$$I(t) = \begin{cases} 0 & t < 0 \\ I_0 & t > 0 \end{cases}$$

a) $$A^\mu(x) = \int d^4y \, \frac{\Theta(x^0 - y^0)}{2\pi} \, \delta\left((x^0 - y^0)^2 - (\vec{x} - \vec{y})^2 \right) J^\mu(y)$$

$$J^\mu(y) = (0, 0, 0, I_0 \Theta(y^0) \delta(y^1) \delta(y^2))$$

$$A^\mu(x) = \int d^4y \, \frac{\Theta(x^0 - y^0)}{4\pi |\vec{x} - \vec{y}|} \, \delta\left((x^0 - y^0) - |\vec{x} - \vec{y}| \right) I_0 \Theta(y^0) \delta(y^1) \delta(y^2) \delta^\mu{}_3$$

$$= \delta^\mu{}_3 \int dy^0 \, dy^3 \, \frac{\Theta(x^0 - y^0) \, \delta\left((x^0 - y^0) - \sqrt{(x^3 - y^3)^2 + (x^2)^2 + (x^1)^2} \right) I_0 \Theta(y^0)}{4\pi \sqrt{(x^3 - y^3)^2 + \vec{x}_\perp^2}}$$

where $\boxed{\vec{x}_\perp^2 = (x^1)^2 + (x^2)^2}$

$$= \delta^\mu{}_3 \int dy^3 \, \frac{I_0 \, \Theta\left(x^0 - \sqrt{(x^3 - y^3)^2 + \vec{x}_\perp^2} \right)}{4\pi \sqrt{(x^3 - y^3)^2 + \vec{x}_\perp^2}}$$

This integral receives contribution only from

$$x^3 - \sqrt{(x^0)^2 - (\vec{x}_\perp)^2} < y^3 < x^3 + \sqrt{(x^0)^2 - (\vec{x}_\perp)^2}$$

due to the Θ function

$$\int \frac{dy^3}{\sqrt{(x^3 - y^3)^2 + \vec{x}_\perp^2}} = -\ln\left[(x^3 - y^3) + \sqrt{\vec{x}_\perp^2 + (x^3 - y^3)^2} \right] + C$$

$$\boxed{\therefore \; A^\mu(x) = \delta^\mu{}_3 \frac{2 I_0}{4\pi} \ln\left[\frac{t + \sqrt{t^2 - (\vec{x}_\perp)^2}}{|\vec{x}_\perp|} \right] \Theta(t - |\vec{x}_\perp|)}$$