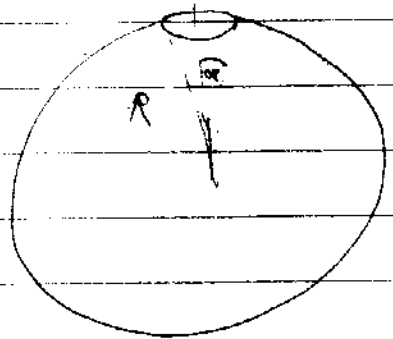


Physics 721 Homework 7 Official Solutions

Problem

1 Jackson 3.2



sigma(theta) = { 0 for 0 <= theta < alpha, Q / (4*pi*R^2) for alpha < theta <= pi }

a)

Inside the sphere,

Phi(r, theta) = sum_l A_l r^l P_l(cos theta)

Using the result on pg. 103 (as explained in lecture),

Phi(r, 0) = 1 / (4*pi) * sum_l A_l r^l * integral dcos theta R^2 sigma(theta) P_l(cos theta) / R^{l+1} = Q * 2pi / (16*pi^2) * sum_l A_l / R^{l+1} * integral_{-1}^{cos alpha} P_l(x) dx

Using

dP_{l+1}(x)/dx - dP_{l-1}(x)/dx = (2l+1) P_l(x)

P_{l+1} |_{-1}^{cos alpha} - P_{l-1} |_{-1}^{cos alpha} = (2l+1) integral_{-1}^{cos alpha} P_l(x) dx

P_{l+1}(-1) = (-1)^{l+1}

1 / (2l+1) [P_{l+1}(cos alpha) - P_{l-1}(cos alpha)] = integral_{-1}^{cos alpha} P_l(x) dx