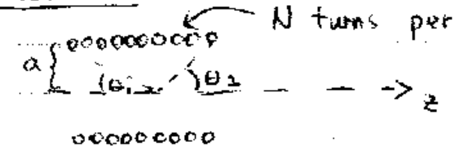
Physics 221 HOMEWORK 9 Official Solutions

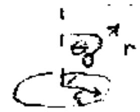
[1] Jackson 5.3



N turns per unit length

To get the field along the $\hat{z}$ axis, it is easier to start fresh from Biot-Savart's law, I will do the problem using the result we derived in lecture (Jackson Eq. 5.36) because it is instructive.

For a single loop, we saw



$$dA_\phi = \frac{dI\,a}{4\pi}\int_0^{2\pi}\frac{\cos\phi'\,d\phi'}{\sqrt{a^2+r^2-2ar\sin\theta_0\cos\phi'}}$$

where one should realize that θ_0 here is <u>not</u> the same as the θ_i in the solenoid diagram.

The B_z field along the axis is B_r at $\theta_0 = 0$ or π

$$dB_r = \frac{1}{r\sin\theta_0}\frac{\partial}{\partial\theta_0}\left(\sin\theta_0\, dA_\phi\right)$$
$$= \frac{1}{r\sin\theta_0}\left[\cos\theta_0\, dA_\phi + \sin\theta_0\frac{d}{d\theta}dA_\phi\right]\Bigg|_{\theta=0}$$

Note that although $dA_\phi \to 0$ as $\theta_0 \to \{0,\pi\}$ because we are dividing by $\sin\theta$, we cannot drop the first term. In fact, we already know from (eq. 5.39) of Jackson that as $\theta_0 \to \{0,\pi\}$,

$$dA_\phi(r,\theta_0) \approx \frac{dI\,a^2 r\sin\theta_0}{4(a^2+r^2)^{3/2}}\left[1+\cdots\right]$$

Hence,

$$dB_z = \frac{\pm\, dI\,a^2}{4(a^2+r^2)^{3/2}} + \frac{\pm\, dI\,a^2}{4(a^2+r^2)^{3/2}}$$

which shows explicitly that one would have made a factor of 2 error if one neglected $dA_\phi|_{\theta=0}$. This is the result (eq. 5.40) of Jackson. Note "+" sign is for $\theta_0 = 0$; "−" is for $\theta_0 = \pi$.