

Lecture 15 (2/21/05)

Electrostatics

8. Variational approach (Jackson 1.12)

Dirichlet:

$$I[\Phi] = \frac{1}{2} \int d^3x (\vec{\nabla} \Phi)^2 - \int d^3x \rho \Phi$$

Neumann:

$$I[\Phi] = \frac{1}{2} \int d^3x (\vec{\nabla} \Phi)^2 - \int d^3x \rho \Phi - \oint_S da f \Phi$$

9. Method of images (Jackson 2.1 - 2.5)

10. Green's function for sphere (Jackson 2.6)

11. Orthogonal functions + expansion (Jackson 2.8)

e.g. periodic over $[\lambda - \frac{a}{2}, \lambda + \frac{a}{2}]$

$$f(x) = \frac{A_0}{2} + \sum_{m=1}^{\infty} [A_m \cos(\frac{2\pi m(x-\lambda)}{a}) + B_m \sin(\frac{2\pi m(x-\lambda)}{a})]$$

$$A_m = \frac{2}{a} \int_{\lambda-a/2}^{\lambda+a/2} dx \cos(\frac{2\pi m(x-\lambda)}{a}) f(x)$$

$$B_m = \frac{2}{a} \int_{\lambda-a/2}^{\lambda+a/2} dx \sin(\frac{2\pi m(x-\lambda)}{a}) f(x)$$

12. Separation of variables (Jackson 2.9)