Lecture 16 (2/23/05)

Electrostatics

13. Conformal mapping

$$z = x + iy$$

$$W(z) = U(z) + iV(z)$$

$$\frac{\partial U}{\partial x} = \frac{\partial V}{\partial y}$$

$$\frac{\partial V}{\partial x} = -\frac{\partial U}{\partial y}$$

a) Harmonic:

$$\vec{\nabla}^2 U = [\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}]U = 0$$

$$[\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}]V = 0$$

b)

$$\vec{\nabla} U \cdot \vec{\nabla} V = 0$$

c)

$$\begin{aligned} \Delta\theta_z &= \frac{1}{i} \ln \frac{\Delta z_1^* \Delta z_2}{|\Delta z_1||\Delta z_2|} \\ &= \Delta\theta_W \end{aligned}$$

d) If $(\frac{\partial^2}{\partial U^2} + \frac{\partial^2}{\partial V^2})f(W) = 0$, then $(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2})f(W(z)) = 0$