Lecture 19 (3/2/05)

Electrostatics

21. Laplace equation in cylindrical coordinates (Jackson 3.7)

$$\Phi = R(\rho)Q(\phi)Z(z)$$

$$\frac{d^2Z}{dz^2} = k^2 Z$$

$$\frac{d^2Q}{d\phi^2} = -\nu^2 Q$$

$$\frac{d^2R}{d\rho^2} + \frac{1}{\rho}\frac{dR}{d\rho} + (k^2 - \frac{\nu^2}{\rho^2})R = 0$$

$$R = \text{linear combination of } J_\nu \text{ and } N_\nu$$

$$J_\nu(x) = (\frac{x}{2})^\nu \sum_{j=0}^{\infty} \frac{(-1)^j}{j!\Gamma(j+\nu+1)}(\frac{x}{2})^{2j}$$

$$N_\nu(x) = \frac{J_\nu(x)\cos\nu\pi - J_{-\nu}(x)}{\sin\nu\pi}$$

$$I_\nu(x) = i^{-\nu} J_\nu(ix)$$

$$K_\nu(x) = \frac{\pi}{2} i^{\nu+1}[J_\nu(ix) + iN_\nu(ix)] \equiv \frac{\pi}{2} i^{\nu+1} H_\nu^{(1)}(ix)$$

$$\Gamma(x) = \int_0^\infty dt e^{-t} t^{x-1}$$

$$\Gamma(n+1) = n!$$

$$\Gamma(n+1) = n\Gamma(n)$$