Lecture 22 (3/9/05)

Multipoles and Dielectrics

2. Energy Density

$$W = \int \rho \Phi(\vec{x}) d^3x$$

where $\Phi(\vec{x})$ =external potential.

$$\Phi(\vec{x}) = \Phi(0) + \vec{x} \cdot \vec{\nabla}\Phi(0) + \frac{1}{2}\sum_{ij} x_i x_j \frac{\partial^2 \Phi(0)}{\partial x_i \partial x_j} + ...$$

$$\vec{E} = -\vec{\nabla}\Phi$$

$$W = q\Phi(0) - \vec{p} \cdot \vec{E}(0) - \frac{1}{6}\sum_{ij} Q_{ij} \frac{\partial E_j(0)}{\partial x_i} + ...$$

3. Electrostatics w/ ponderable media (Jackson 4.3)

$$\vec{\nabla} \cdot \vec{D} = \rho$$

$$\vec{D} = \vec{E} + \vec{P}$$

$$\vec{P} = \chi_e \vec{E}$$

$$\vec{D} = \epsilon \vec{E}$$

$$\epsilon = 1 + \chi_e$$

$$(\vec{D}_2 - \vec{D}_1) \cdot \vec{n}_{21} = \sigma$$

$$(\vec{E}_2 - \vec{E}_1) \times \vec{n}_{21} = 0$$

$\vec{n}_{21}$ = unit normal directed from region 1 to region 2