Lecture 25 (3/16/05)

Magnetostatics

5. Vector Potential and Magnetic induction for a circular current loop (Jackson 5.5)

Complete elliptic integrals:

$$K(k) = \int_0^{\pi/2} \frac{d\phi}{\sqrt{1 - k^2 \sin^2 \phi}}$$

$$E(k) = \int_0^{\pi/2} d\phi \sqrt{1 - k^2 \sin^2 \phi}$$

6. Magnetic moment:

$$\mathcal{M} = \frac{1}{2}[\vec{x} \times \vec{J}(\vec{x})]$$

$$\vec{m} = \int d^3x' \mathcal{M}(\vec{x}')$$

$$\vec{B}(\vec{x}) = \frac{1}{4\pi} \left[\frac{3\hat{x}(\hat{x} \cdot \vec{m}) - \vec{m}}{|\vec{x}|^3} \right]$$

planar current loop:

$$\begin{aligned} \vec{m} &= \frac{I}{2} \oint \vec{x} \times d\vec{l} \\ &= I \times \text{area} \end{aligned}$$

7. Leading term in force and torque of a localized current distribution in an external magnetic field

$$\vec{F} = \vec{\nabla}(\vec{m} \cdot \vec{B})$$

$$\vec{N} = \vec{m} \times \vec{B}(0)$$

energy:

$$U = -\vec{m} \cdot \vec{B}$$