

Lecture 27 (3/28/05)

Magnetostatics

11. Ferromagnetic material in an external magnetic field

12. Magnetic shielding

13. Faraday law of induction

$$\oint \vec{E} \cdot d\vec{l} = -\partial_t \int \vec{B} \cdot d\vec{a}$$

14. Energy in a magnetic field

$$\delta W = \int \delta \vec{A} \cdot \vec{J} d^3x$$

$$\delta W = \int \delta \vec{B} \cdot \vec{H} d^3x$$

linear material

$$W = \frac{1}{2} \int \vec{J} \cdot \vec{A} d^3x$$

$$W = \frac{1}{2} \int \vec{H} \cdot \vec{B} d^3x$$

energy of an object of permeability μ_2 in a medium of permeability μ_1 and \vec{B}_1 :

$$\Delta W = \frac{1}{2} \int_{V_2} (\vec{B} \cdot \vec{H}_1 - \vec{H} \cdot \vec{B}_1) d^3x$$

15. Magnetic Inductance

$$\begin{aligned} W &= \frac{1}{2} \int \vec{J} \cdot \vec{A} d^3x \\ &= \frac{1}{8\pi} \sum_{i=1}^N \int d^3x_i \sum_{j=1}^N \int d^3x'_j \frac{\vec{J}(\vec{x}_i) \cdot \vec{J}(\vec{x}'_j)}{|\vec{x}_i - \vec{x}'_j|} \\ &= \frac{1}{2} \sum_{i=1}^N L_i I_i^2 + \sum_{i=1}^N \sum_{j>i}^N M_{ij} I_i I_j \end{aligned}$$

$$M_{ij} = \frac{1}{4\pi I_i I_j} \int d^3x_i \int d^3x'_j \frac{\vec{J}(\vec{x}_i) \cdot \vec{J}(\vec{x}'_j)}{|\vec{x}_i - \vec{x}'_j|}$$