Lecture 5 (1/28/05)

Relativsitic Electrodynamics (continued)

12. Vector Potential + gauge invariance (Jackson 6.2)

* Any 3-vector can be written locally as the gradient of a scalar added to a curl of a vector.

* From Poincare Lemma, locally, curlfree vector field is a gradient and a divergencefree vector field is a curl.

13. Transformation of Electromagnetic field (Jackson 11.10)

14. Relativistic Lagrangian and Hamiltonian (Jackson 12.1B)

Jackson calls the action A. Here we use S.

Functional differentiation

formally:

$$\frac{\delta S[f(\lambda)]}{\delta f(\lambda_1)} = \lim_{\epsilon \to 0} \frac{S[f(\lambda) + \epsilon\delta(\lambda_1 - \lambda)] - S[f(\lambda)]}{\epsilon}$$

e.g.

$$I[f(t)] = \int dt \sin f(t)$$

$$\frac{\delta I}{\delta f(t)} = \cos f(t)$$

action:

$$S[q(\lambda)] \equiv \int d\lambda L(q(\lambda), \frac{d}{d\lambda} q(\lambda))$$

equation of motion:

$$\frac{\delta S}{\delta q(\lambda)} = 0$$

Hamiltonian (time independent):

$$L = L(q, \frac{d}{d\lambda} q)$$

$$p = \frac{\partial L}{\partial (\frac{d}{d\lambda} q)}$$

$$H(p, q) = p(\frac{d}{d\lambda} q) - L$$

$$\frac{\partial H}{\partial p} = \frac{d}{d\lambda} q$$

$$\frac{\partial H}{\partial q} = -\frac{d}{d\lambda} p$$