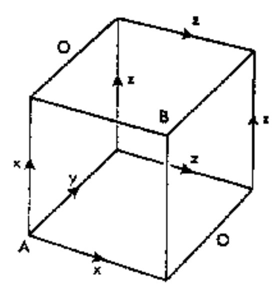
3

ELECTROMAGNETISM

1. From the symmetry of the cube the currents in the resistors must be as shown in the figure. Conservation of current at the corners requires:

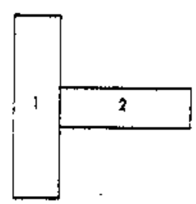
$$I = 2x + y, \qquad \text{and} \qquad y = 2z,$$



where I is the input current. The requirement that the voltage between A and B be independent of path yields the additional equation $2xR = 2(y + z)R$. These three equations have the solution $x = 3I/8$; $y = I/4$; and $z = I/8$. The resistance between A and B is $(2xR/I)$; therefore $R_{AB} = 3R/4$.

2. If 1 A is fed into A and taken out at infinity, then from symmetry, $\frac{1}{4}$ A will flow in AC. Likewise if 1 A is taken from C and fed in at infinity, $\frac{1}{4}$ A will flow in AC. By the superposition of the two solutions, we obtain the solution to the given problem with the current in AC being $\frac{1}{2}$ A.

3. Place the bars as shown at the right: If 1 is magnetized and 2 is not, there will be no net attraction between the magnets, by symmetry. If 2 is magnetized and 1 is not, the bars will attract one another, because of the poles induced in 1 by the field of 2.



4. Due to the linearity of the equations of electrostatics, one has $q_c = C_1V$, and $q_p = C_2V$ for the charges on the conductor and plate respectively when they are in contact, and V is their common voltage. Thus

$$q_c/q_p = C_1/C_2 = \text{const.}$$

After the first contact $q_c = q$ and $q_p = Q - q$, and ultimately $q_p \to Q$; hence $q_c \to Qq/(Q - q)$.

5. The battery supplies the constant power, $P = EI$. The electrostatic energy of the capacitor, $U = qE/2$, is changing at the rate

$$\frac{dU}{dt} = \frac{E}{2}\frac{dq}{dt} = \frac{EI}{2}.$$

Thus the battery is doing twice as much work as is being stored in the capacitor. The difference appears as work done by the capacitor on the external agent that is causing the capacitance to change.

6. Let Q be the charge on one plate; let σ be the charge density on this plate surface; let I, J be the interplate current and current density respectively; let V be the voltage between the plates.

$$\frac{V}{R} = I = \int \mathbf{J}\cdot d\mathbf{A} = g\int \mathbf{E}\cdot d\mathbf{A} = \frac{g}{\epsilon}\int \sigma dA \qquad \text{(Gauss' Law)}$$

$$= \frac{gQ}{\epsilon} = \left(\frac{g}{\epsilon}\right)CV.$$

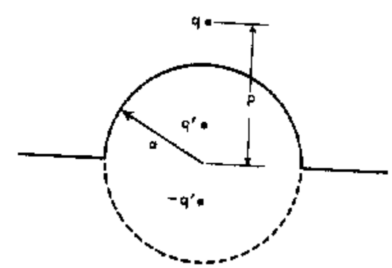
Hence $RC = \epsilon/g$.

7. The linearity of Maxwell's equations allows us to think of the magnetic field as arising from two current densities:

(1) A current density $j = I/\pi(b^2 - a^2)$, carried by the cylinder of radius b, and

(2) a current density $-j$ carried by a cylinder of radius a.

The sum of the current densities (1) and (2) is the current distribution of the bored-out cylinder. From Ampère's circuital law $\oint \mathbf{H}\cdot d\mathbf{l} = (4\pi/c)\int \mathbf{j}\cdot d\mathbf{A}$, one finds that (1) produces a magnetic field $H = 2Id/c(b^2 - a^2)$ at the center of the hole, while (2) produces no magnetic field at the center of the hole. The resultant magnetic field, H, is thus given by $H = 2Id/c(b^2 - a^2)$.



8. The solution is by the method of images. We choose image charges inside the conductor, with location and strength so as to make the conductor an equipotential. To make the surface of the boss equipotential, we must place an image $q' = -qa/p$ at distance a^2/p from the origin, on the line joining