Lecture 19

Note Title 11/5/2007

Example of an inflationary scenario.

"Free" scalar field

Consider a real scalar field.

$$V(\phi) = \frac{1}{2} m^2 \phi^2 \qquad \text{(one parameter model)}$$

$$\varepsilon \equiv \frac{M_{Pl}^2}{16\pi}\left(\frac{V'(\phi)}{V(\phi)}\right)^2 \approx \frac{M_{Pl}^2}{16\pi}\left(\frac{2}{\phi}\right)^2 \ll 1 \qquad \text{(sufficient negative pressure)}$$

$$\eta \equiv \frac{M_{Pl}^2}{8\pi}\left(\frac{2}{\phi^2}\right) \ll 1 \qquad \text{(slow roll)}$$

$$\therefore \quad \boxed{\phi \gg M_{Pl}} \qquad \text{satisfies both of these conds.}$$

60 e-foldings

$$H^2 = \frac{8\pi}{3M_{Pl}^2} V(\phi)$$

$$N = \int H\,dt$$

$$= \sqrt{\frac{8\pi}{3}}\frac{1}{M_{Pl}}\int \sqrt{V}\,dt = \sqrt{\frac{8\pi}{3}}\frac{1}{M_{Pl}}\int \frac{\sqrt{V}}{\dot\phi}\,d\phi$$

$$= \sqrt{\frac{8\pi}{3}}\frac{1}{M_{Pl}}\int \frac{\sqrt{V}\,(-3H)}{V'(\phi)}\,d\phi = \sqrt{\frac{8\pi}{3}}\frac{-3}{M_{Pl}}\int \frac{\sqrt{V}}{V'(\phi)}\sqrt{\frac{8\pi}{3}}\frac{1}{M_{Pl}}\sqrt{V}\,d\phi$$

$$= -\frac{8\pi}{M_{Pl}^2}\int_{\phi_i}^{\phi_f}\frac{V(\phi)}{V'(\phi)}\,d\phi = -\frac{8\pi}{M_{Pl}^2}\int_{\phi_i}^{\phi_f}\frac{d\phi}{\sqrt{\varepsilon}}\frac{M_{Pl}}{4\sqrt{\pi}}$$

$$= \frac{-2\sqrt{\pi}}{M_{Pl}}\int_{\phi_i}^{\phi_f}\frac{d\phi}{\sqrt{\varepsilon}} = \frac{-\cancel{2}\sqrt{\pi}}{M_{Pl}}\int_{\phi_i}^{\phi_f}\frac{d\phi}{\left(\frac{\cancel{2}}{\phi}\right)\frac{M_{Pl}}{4\sqrt{\pi}}} = \boxed{\frac{-2\pi}{M_{Pl}^2}\left(\phi_f^2 - \phi_i^2\right)}$$

For $\phi_f \ll \phi_i$,

$$\phi_{60} \sim \sqrt{10}\, M_{Pl}$$

Density perturbations constraint:

$$\sqrt{P_R(k)} = \sqrt{\frac{V}{24\pi^2\epsilon}\,\frac{(8\pi)^2}{M_{Pl}^4}} \sim 10^{-5}$$

$$\frac{1}{M_{Pl}^2}\sqrt{\frac{\frac{1}{2}m^2\phi_i^2\,(8\cancel{\pi})^2}{24\cancel{\pi^2}\left(\frac{M_{Pl}^2}{16\pi}\right)\left(\frac{2}{\phi_i}\right)^2}} \sim 10^{-5}$$

$$\frac{m\phi_i}{M_{Pl}^2}\left(\frac{4\sqrt{\pi}}{\sqrt{3}}\right) \sim 10^{-5}$$

This number translates into tuning

using $\frac{\phi_i}{M_{Pl}} \sim \sqrt{10}$, we have $\boxed{\therefore\ m \sim 10^{-6} M_{Pl}}$

Why is this a problem?

EFT: All operators not forbidden by symmetries should be present. These require small/tuned coeffs.

term	classical mass contribution	quantum mass
$\lambda\phi^4$	$\sim \lambda\phi_i^2 \sim (\lambda 10)M_{Pl} \Rightarrow \lambda < 10^{-12}$	$\sim \frac{\lambda}{16\pi}\Lambda^2 \Rightarrow \lambda < 10^{-11}$ if $\Lambda \sim M_{Pl}$
$c\frac{\phi^6}{\Lambda^2}$	$\sim c\frac{\phi_i^4}{\Lambda^2} \sim c10^2 M_{Pl}^2$ if $\Lambda \sim M_{Pl}$ $\Rightarrow c < 10^{-14}$	$\sim \frac{c10M_{Pl}^2\Lambda^2}{\Lambda^2\,16\pi} \Rightarrow c < 10^{-12}$
$\vdots$		

After fitting the amplitude of the spectrum to data, there is a prediction for the shape:

$$\Delta^2_R(k) = \Delta^2_R(k_c)\left(\frac{k}{k_c}\right)^{n_s(k)-1}$$

$$n_s(k) - 1 \approx 2\eta - 6\varepsilon = 2\left(\frac{M_{Pl}^2}{8\pi}\right)\left(\frac{2}{\phi^2}\right) - 6\,\frac{M_{Pl}^2}{16\pi}\left(\frac{2}{\phi}\right)^2$$

$$= \frac{-M_{Pl}^2}{\pi\phi^2}$$

To map ϕ to k, we have been using

$$dN \approx \frac{-2\sqrt{\pi}}{M_{Pl}}\frac{d\phi}{\sqrt{\varepsilon}}$$

As you will show in your homework,

$$d\ln k \approx (1-\varepsilon)\,dN$$

$$\therefore \quad d\ln k \approx -\frac{(1-\varepsilon)2\sqrt{\pi}}{M_{Pl}\sqrt{\varepsilon}}\,d\phi = \frac{-\left(1 - \frac{M_{Pl}^2}{4\pi\phi^2}\right)4\pi\phi}{M_{Pl}^2}\,d\phi$$

$$\boxed{k(\phi) \sim k_i \exp\left(\frac{2\pi}{M_{Pl}^2}\left[\phi_i^2 - \phi^2\right]\right)}$$

Small movement in $\phi \Rightarrow$ large change in k