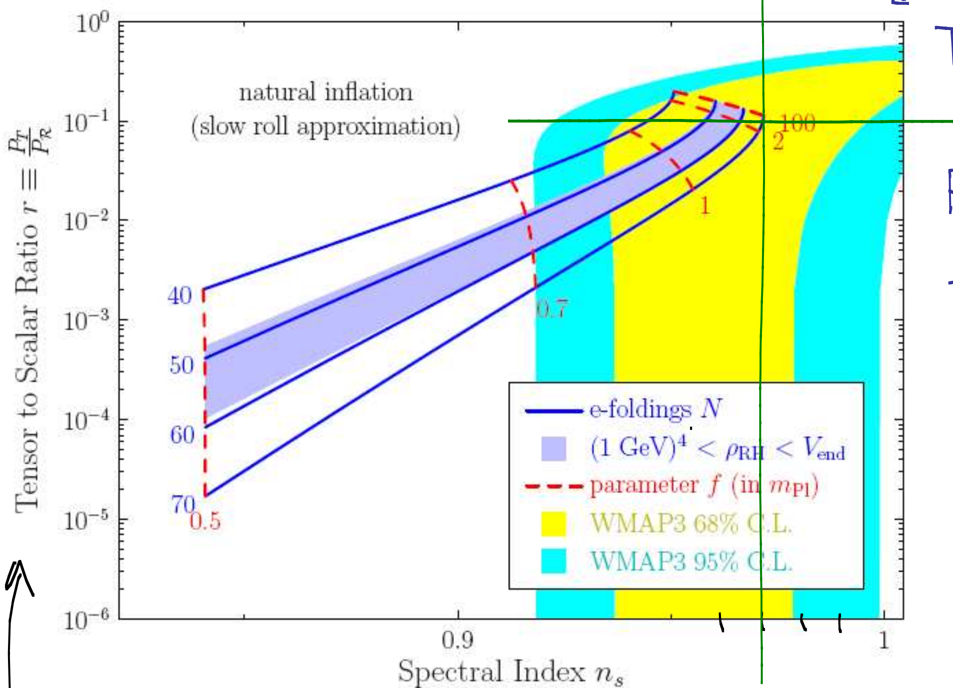
Lecture 20



Taken from hep-ph/0609144

For $V = \frac{1}{2} m^2 \phi^2$ we had found in lecture 19

$n_s - 1 \approx \frac{-M_{Pl}^2}{\pi \phi^2}$ which for $N \approx 60$ corresponded to

$\phi_{60} \approx \sqrt{10}\, M_{Pl} \Rightarrow n_s - 1 \approx \boxed{-0.03}$

$r \equiv \frac{P_T}{P_R}$ ← tensor perturbations have not been measured yet.

As we will show today, $r \approx 16 \varepsilon$

$$\approx \left. \frac{4 M_{Pl}^2}{\pi \phi^2} \right|_{\phi_{60}} \approx \boxed{0.1}$$

Tensor perturbation generation (gravity wave production)

As you will see, although the equations for tensor perturbation generated from quantized equations are similar to those for scalar perturbations, the physics is a bit different.

i.e. $\langle \delta\rho_S \rangle = 0$ for scala perturbations (by the way, what is a scalar perturbation?) $\leftrightarrow$ particle production!

$\langle \delta\rho_T \rangle \neq 0$ $\leftrightarrow$ honest particle production!

$$S = -\frac{1}{16\pi G}\int d^4x \sqrt{g}\, R$$

$$ds^2 = a^2(\eta)\left(-d\eta^2 + (\delta_{ij} + h_{ij})\, dx^i dx^j\right)$$

recall lecture 6

latin indices raised and lowered using δ_{ij}

h_{ij} satisfies $\quad h^i{}_i = 0 \qquad h_{ij}{}^{|j} = 0$

Again expand S to quadratic order in h_{ij}

$$S_T = -\frac{1}{64\pi G}\int d^4x\,(a^2)\left(\partial_\eta h^i{}_k\, \partial_\eta h^k{}_i - \vec{\nabla} h^i{}_k \cdot \vec{\nabla} h^k{}_i\right)$$

Quantize this to compute correlation function for $h^i{}_k$.

Note $\quad -\frac{1}{64\pi G}\partial_\eta h^i{}_k\, \partial_\eta h^k{}_i = -\frac{1}{16\pi G}\,\frac{1}{2}\,\frac{1}{2}\,\partial_\eta h^i{}_k\, \partial_\eta h^k{}_i$

$\Rightarrow \; h^i{}_k \frac{1}{\sqrt{16\pi G}}$ analogous to a scalar field φ since 2 polarizations!

Heisenberg

$$\partial_\eta^2 h^i{}_j + 2\left(\frac{\partial_\eta a}{a}\right)\partial_\eta h^i{}_j - \nabla^2 h^i{}_j = 0$$

$$h^i{}_j = a^{-1}(\eta)\int \frac{d^3k}{(2\pi)^{3/2}}\left[a_k^{(s)} e^{(s)i}{}_j h(\eta) + a^\dagger_{-k} e^{(s)i}{}_j{}^* h^*(\eta)\right] e^{i\vec{k}\cdot\vec{x}}$$

$$\partial_\eta(a^{-1}h) = -\frac{a'}{a^2}h + \frac{1}{a}h'$$

$$\partial_\eta^2(a^{-1}h) = -\frac{a''}{a^2}h + \frac{2a'^2}{a^3}h - 2\frac{a'}{a^2}h' + \frac{h''}{a}$$

$$-\frac{a''}{a^2}h + \cancel{\frac{2a'^2}{a^3}h} - \cancel{2\frac{a'}{a^2}h'} + \frac{h''}{a} + 2\frac{a'}{a}\left(\cancel{-\frac{a'}{a^2}h} + \cancel{\frac{1}{a}h'}\right) + k^2\frac{h}{a} = 0$$

$$\therefore \quad \boxed{h'' + \left(k^2 - \frac{a''}{a}\right)h = 0}$$

Since $a \approx -\frac{1}{H\eta}$ $\qquad \frac{a''}{a} \approx \frac{2}{\eta^2}$

$$h'' + \left(k^2 - \frac{2}{\eta^2}\right)h = 0$$

With boundary cond, $h \xrightarrow[\eta\to-\infty]{} \frac{e^{-ik\eta}}{\sqrt{2k}}$, we find

$$h_k = \frac{\sqrt{\pi}}{2}\sqrt{-\eta}\, H_\nu^{(1)}(-k\eta)\, e^{i\frac{\pi}{2}(\nu+\frac{1}{2})} \qquad \nu = \frac{\sqrt{9}}{2} = \frac{3}{2}$$

Now, a common definition of the tensor power spectrum is $\Delta^2_{h_{ij}}(k) \equiv \dfrac{2k^3}{2\pi^2}\left\langle |h_+|^2 + |h_\times|^2 \right\rangle$

(Note the extra factor of 2 compared to scalar power spectrum: $\Delta^2_{R}(k) \equiv \dfrac{k^3}{2\pi^2}|R_k|^2$ from Lecture 17)

Since $H^{(1)}_{\nu}(z) \underset{z\to 0}{\sim} -i\left(\dfrac{2}{z}\right)^{\nu}\dfrac{\Gamma(\nu)}{\pi}$

convention

Normalization (see pg. 2)

$$\Delta^2_{h_{ij}}(k) \approx 2 \cdot 2\,\frac{k^3}{2\pi^2}\left(\frac{2}{-k\eta}\right)^{2\nu}\frac{(\Gamma(\nu))^2}{\pi^2}\,\frac{\pi}{4}(-\eta)\,\frac{16\pi G}{a^2}$$

$h^2 \to h^2_{ij}$

2 polarizations

$$= \frac{4}{(2\pi^2)}\left(\frac{2}{-\eta}\right)^{3}\frac{(\Gamma(\nu))^2}{\pi^2}\,\frac{\pi}{4}(-\eta)\,\frac{16\pi G}{a^2}$$

Compare w/ results of Lecture 18:

Again scale invariant to this level of approx.

$$\therefore \quad \frac{\Delta^2_h}{\Delta^2_R} \approx 4\,\frac{\cancel{M^2_{Pl}}\,\varepsilon}{4\pi}\,\frac{16\pi}{\cancel{M^2_{Pl}}} = \boxed{16\varepsilon}$$

Hence, we have justified the statement on pg. 1.