Physics 801 Homework 1:
(Due: 9/20/07)

1. (long problem) Derive the Friedmann equation starting from Einstein equations:

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi G T_{\mu\nu}$$

with the FRW metric ansatz

$$ds^2 = -dt^2 + a^2(t)\left(\frac{dr^2}{1 - Kr^2} + r^2 d\Omega^2\right)$$

and

$$T^{\mu}_{\ \nu} = \begin{pmatrix} -\rho(t) & & & \\ & P(t) & & \\ & & P(t) & \\ & & & P(t) \end{pmatrix}.$$

2. Show that a 3-sphere defined by

$$\sum_{i=1}^{4} (x^i)^2 = \alpha^2$$

embedded in Euclidean $4D$ space has a metric on it which can be expressed as

$$ds^2 = \frac{dr^2}{1 - f(\alpha) r^2} + r^2 d\Omega^2$$

where $f(\alpha)$ is a function of α. [Note that r^2 here is defined as $r^2 = (x^1)^2 + (x^2)^2 + (x^3)^2$ and the metric on the sphere is $ds^2 = \sum_{i=1}^{4} (dx^i)^2$.]

3. Compute the stress tensor of the field $\chi_{\mu\nu}$ where $\chi_{\mu\nu} = -\chi_{\nu\mu}$ and has the action

$$S = \int d^4x \sqrt{g} [\partial_{[\alpha} \chi_{\mu\nu]} \partial_{[\beta} \chi_{\lambda\kappa]} g^{\alpha\beta} g^{\lambda\mu} g^{\kappa\nu} + m^2 \chi_{\mu\nu} \chi^{\mu\nu}]$$

where $[...]$ is the antisymmetrizer:i.e.

$$\partial_{[\alpha} \chi_{\mu\nu]} = \frac{1}{6} (\partial_\alpha \chi_{\mu\nu} - \partial_\mu \chi_{\alpha\nu} - \partial_\nu \chi_{\mu\alpha} + \partial_\mu \chi_{\nu\alpha} + \partial_\nu \chi_{\alpha\mu} - \partial_\alpha \chi_{\nu\mu}).$$

[You do not have to solve any equation of motion. Just compute the stress tensor in terms of the metric and the field $\chi_{\mu\nu}$.]

4. For a Hubble constant of $H_0 = 70$km/(sMpc), plot the age of a matter dominated universe as a function of Ω_0.