but most of the quantitative results should be considered with some reserve, perhaps as best estimates.

In spite of their uncertainties, judiciously considered quantitative estimates are interesting and important. Whatever their uncertainties, they often supplant much weaker, and sometimes erroneous, qualitative insights. Consequently, I have attempted to provide numerical values almost everywhere: sometimes when the results are somewhat uncertain, sometimes when the numbers are quite trivial but not necessarily immediately accessible to the reader.

As this exposition is directed toward those interested in baseball, not physics, I have chosen to present quantitative matters in terms of familiar units. Hence, I use the English system of measures—distances in feet and inches, velocities in miles per hour (mph), and forces in terms of ounce and pound weights. Moreover, I have often chosen to express effects on the velocities of batted balls in terms of deviations of the length of a ball batted 400 feet (likely to be a long home run) under standard conditions.

To express the goals of this book, I can do no better than to adopt a modification of a statement from Paul Kirkpatrick’s article “Batting the Ball”: The aim of this study is not to reform baseball but to understand it. As a corollary to this statement of purpose, I must emphasize that the book is not meant as a guide to players; for all of the ways to learn to better throw and bat a ball, an academic study of the mechanics of the actions must be the least useful.

THE FLIGHT OF THE BASEBALL

THE BASEBALL—AIR RESISTANCE

From the Official Baseball Rules: 2001:

1.09 The ball should be a sphere formed by yarn wound around a small sphere of cork, rubber, or similar material covered with two stripes of white horsehide or cowhide, tightly stitched together. It shall weigh not less than 5 nor more than 5 ⅛ ounces avoirdupois and measure no less than 9 nor more than 9 ⅛ inches in circumference.

The description of the baseball in the rule book, ingenuous and charming, is not that of an engineer; the manufacturer (once in Chicopee, Massachusetts, then Haiti, then Taiwan, and now, at the beginning of the third millennium, in Costa Rica) is given these further directions: “The cork-rubber composite nucleus, enclosed in rubber, is wound with 121 yards of blue-gray wool yarn, 45 yards of white wool yarn, and 150 yards of fine cotton yarn. Core and winding are enclosed by rubber cement and a two piece cowhide—horsehide before 1974—cover hand-stitched together with just 216 raised red cotton stitches.”
Much more is required to completely define the ball that is the center of the sport of baseball, but its flight is largely determined by the size and weight constraints listed in the rules. The paths of baseballs projected at velocities common to the game are strongly influenced by air resistance. As the ball passes through the air, it pushes the air aside and loses energy, and thus velocity, through the work it does on the air. The forces on the ball from the resistance of the air are typically of the same magnitude as the force of gravity. A ball batted with an initial velocity of 110 mph at an angle of 35° from the horizontal would go about 750 feet in a vacuum; at Shea Stadium in New York, it will travel only about 400 feet. Hence, it is necessary to understand the fluid dynamics of air flow around spheres to understand the flight of a baseball.

When an object (such as a baseball) passes through a fluid (such as air), the fluid affects the motion of the object as it flows about that object. Moreover, for all fluids and all objects, the character of the flow of the fluid is largely determined by the value of a (dimensionless) Reynolds number proportional to the density of the fluid, the fluid velocity, and the size of the object, and inversely proportional to the viscosity of the fluid. For a given Reynolds number, the behavior of the gaseous fluid of stars—interacting with each other through gravity—that make up a galaxy a hundred thousand light-years across is described in very much the same way as the behavior of the molecules of air passing through an orifice 1 micron across, where a micron is about equal to the resolution of a high-power microscope.

The most interesting actions in the game of baseball take place when velocities of the ball range from a few miles per hour (and Reynolds numbers of 10,000) to values near 120 mph (and Reynolds numbers near 200,000). For velocities in that range below about 50 mph, the flow of the air around the ball is rather smooth, though trailing (Von Karman) vortices are generated. This airflow does not actually reach the surface of the ball where there is a quiet (Prandtl) boundary layer. A very, very small insect (perhaps a plant aphid) sitting on the moving ball would feel no breeze at all. At velocities above 200 mph the flow penetrates the boundary layer (the aphid would have to hold on very tightly to avoid being blown off) and the air at the boundary—and trailing behind the ball—is quite turbulent. I label the two regions conveniently (if a little inaccurately) as smooth and turbulent.

Hence, for a baseball passing through air at a velocity less than 50 mph the airflow is smooth, while the airflow is turbulent for velocities greater than 200 mph. But much of the subtlety of baseball is derived from the fact that so much of the game is played in the region between definitely smooth flow and definitely turbulent flow, at ball velocities greater than 50 mph and less than 120 mph. For balls traveling at the transition velocities between 50 and 120 mph, the flow can be smooth or turbulent, depending on the detailed character of the surface of the ball and its motion. By and large, turbulence will be induced at lower velocities by roughness in the surface, and held off to higher velocities if the surface is very smooth. Surprisingly, at a given velocity the air resistance is less for turbulent flow than for smooth flow. It seems that at low velocities the ball, with its boundary layer of still air, is effectively larger than it is at higher velocities with the boundary layer blown off and thus the higher-velocity ball moves a smaller column of air.

From our understanding of fluid flow, it is convenient to describe the drag or retarding force on a moving baseball (or equivalent sphere) as proportional to the cross-sectional area of the ball, as a larger ball must push more air out of the way. The force is also proportional to the square of the velocity of the ball (doubling the velocity increases the drag by a factor of four). A ball with double the velocity must push twice as much air out of its way, and that air will be pushed twice as hard. If the air is less dense, it is easier to push away; hence the drag is also proportional to the density of the air. Consequently, the drag varies to some extent with temperature and altitude, just as the air density varies with those factors. As I have mentioned, the character of the airflow around the ball can change with velocity, and such
changes affect the resisting drag force also. We take that into account by a further proportionality of the drag to a "drag coefficient" that depends only on the value of the Reynolds number which is proportional to the velocity of the ball.

Figure 2.1 shows an estimate of the variation of the drag coefficient for a baseball as a function of the velocity of the ball. The drag force on the baseball will also depend to some extent upon the orientation of the stitches on the ball. When the ball is rotating—as is usually the case—the drag will depend on the position of the axis of rotation with respect to the stitch pattern of the ball, on the direction of the axis with respect to the ground and the direction of the ball’s flight, and on the velocity of rotation of the ball. The drag on a rapidly spinning ball is probably slightly larger than that on a slowly rotating ball. But that effect must be small; I estimate that the incremental drag on the spinning ball will usually not be much larger than 5 percent of the drag on the nonspinning ball. This would mean that for a 90-mph fastball thrown with a spin of 1500 rpm, the extra drag will reduce the speed of the ball so that it will cross the plate traveling about 0.5 mph slower than a ball thrown with very little spin.

For rotating balls, the dependence of the drag upon these factors is not likely to be large, however, and we can consider that the drag effects described here represent a kind of average over different orientations of the stitch patterns with the spin directions. Aside from these caveats, the values of the coefficients for a baseball are not well known, but wind tunnel measurements have been made of the forces on balls at velocities up to 95 mph that support the solid curve. In particular, wind tunnel measurements showed the ball suspended nearly motionless in an upward-directed 95-mph airstream. Therefore, for a ball moving through the air with a velocity of 95 mph, the drag force is about equal to the weight of the ball. Hence, the terminal velocity of a ball dropped from a great height is but 95 mph.

The broken-curve line in the figure corresponds to values of the drag coefficient that would generate a drag force equal to the force of gravity. The ball suspended in the 95-mph vertical airstream was held steady by the force of the air. This would be the case only if the force is greater if the ball is falling, and hence moving faster through the air, and if the force is weaker if the ball is rising, and thus moving more slowly with respect to the air. Consequently, the observed stability of the ball demands that the solid curve cross the broken line from left to right at 95 mph, further defining the variation of the drag coefficient with the velocity of the baseball.

The values expressed for larger velocities, of up to 150 mph, are estimates, albeit guided by theoretical considerations. The mean uncertainties are perhaps 10 percent for velocities less than 120 mph, which is near the highest velocity reached by a ball in play. The rather gradual reduction in the drag coefficient with increasing velocity, from the value of about 0.5 for velocities less than 50 mph to those of about 0.2 for velocities greater than 120
mph, suggests that the transition from smooth to turbulent flow of the air passing the baseball in flight occurs gradually.

The values of the drag coefficient for an ideally smooth ball and a uniformly rough ball — about as rough as a ball completely covered with stitches — are shown also. The variation of the drag coefficient with velocity will have the same general character for a ball that is uniformly a little rougher or a little smoother, but the drag minimum will be found at lower velocities for a rougher ball and at higher velocities for the smoother one.*

At the velocities of 50 to 120 mph dominant in baseball, the air passes over a smooth ball the size of a baseball in a smooth high-resistance flow; turbulence is not induced until velocities approach 200 mph. However, a real baseball, roughened by its raised-stitch patterns, induces low-resistance turbulent flow at baseball velocities. Consequently, if the baseball were quite smooth rather than laced with protuberant stitches, as is the case, it could not be thrown or batted nearly as far as in fact is — a stitched baseball batted 400 feet would travel only about 350 feet, if the ball were very smooth. This effect is dramatic in golf; the air resistance from the smooth flow about a smooth ball is so great that the ball goes nowhere. The ball is therefore artificially roughened by the dimples impressed in the covering to induce turbulence and reduce air resistance.

At velocities near 175 mph, where the resistive force for smooth balls falls off sharply, the resistance on the ball actually becomes smaller as the velocity of the ball increases. The resistance on a smooth ball the size of a baseball traveling 190 mph is less than the resistance on such a ball traveling 160 mph. This sharp dip in the drag coefficient at the onset of turbulence has been called the “drag crisis.” For uniformly rough balls the sharp reduction in drag with increasing velocity comes at lower velocities; the rougher the ball the lower the velocity at which turbulence is induced and the lower velocity of the drag crisis. If a baseball suffered such a drag crisis at velocities typical of the game, the anomalous effects could be important.* For example, a ball hit against the wind might go farther than a ball hit with the wind. It seems, however, that the baseball, usually rotating in flight and thus presenting different configurations of smoothness and roughness to the air as a consequence of the changing orientation of its stitches, is by no means a uniformly rough ball and therefore does not undergo the sudden transition from smooth to turbulent flow that characterizes a drag crisis.

Since the drag for a rough ball can be less than that for a smoother ball, even if there is no drag crisis for a baseball the distance a batted ball will travel might still depend upon the character of its surface. A very rough, scarred ball with a surface that could induce turbulence at low velocities could well travel farther than a new, smooth ball. Smaller changes could also be significant. Such a change in the surface of the ball may have occurred in 1974, when the traditional horsehide cover was replaced by cowhide. However, a judicious estimate suggests that for the range of surface conditions tolerated for baseballs used in the major leagues, the dependence of the drag on the character of the basic surface skin can be neglected; the skins of cows and horses are not that different. Still, any significant changes in the height of the stitches might change the velocity at which turbulence begins and thus affect the flight of the ball and the distance it can be hit.

Returning to the regulation baseball, the solid curve of Figure 2.2 shows the variation of the drag force on the ball with velocity derived from the baseball drag coefficients of Figure 2.1. The force is expressed in terms of the weight of the ball; hence, the value 1.0 of the ordinate corresponds to the force of gravity. Again, there may be significant differences in the drag force for different orientations of the stitch configurations with respect to

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*These properties of balls of various degrees of roughness were established experimentally by E. Aschenbach and published in 1974 in the Journal of Fluid Mechanics.

the direction of motion, hence, the forces shown here must be considered as a kind of average over the range of possible stitch orientations.

**SPIN AND THE MAGNUS COEFFICIENT**

Interest in the left-right curvature of balls sailing through the air is probably as old as ball games themselves. Isaac Newton, at the age of twenty-three, discussed the curvature of court tennis balls—a little like small baseballs—in terms that make good sense today. In the nineteenth-century genesis of mathematical physics, Lord Rayleigh analyzed the curvature of the path of spinning balls and P. G. Tait, an eminent Scottish physicist, wrote extensively on the curves of golf balls—perhaps in an attempt to understand and cure a slice.

The total force on a baseball, from the normal air pressure of 14.6 pounds per square inch, pushing the ball toward third base as it travels from pitcher to batter, is nearly 100 pounds. Of course, there is ordinarily a nearly identical force pushing the ball toward first base. If these forces differ by as much as an ounce and a half, or about one part in a thousand, a ball thrown to the plate at a velocity of 75 mph will be deflected, or curve, a little more than a foot. Such modest, asymmetric force imbalances are generated by a spinning ball and by asymmetric placement of the stitches on the ball.

If the resistive force on a ball is proportional to the square of the velocity of the air passing the ball, it would seem probable that there would be such an unbalanced force on a spinning ball, on an axis normal to the velocity even as the velocity through the air of one side of the ball at the spin equator is greater than the velocity of the other side. Such a force, directed at right angles to the direction of the air velocity and to the axis of spin, has long been known and is usually called the Magnus force.

Some insight into the force imbalance can be gained by considering the different forces that follow from the different velocities of the opposite sides of a spinning ball. Figure 2.3 suggests the origin of this transverse force for the "normal" situation, where the air resistance forces increase as the velocity increases. Consider that a curveball is thrown (from left to right in the figure) by a right-handed pitcher at a speed of 70 mph, so that it rotates 16 times counterclockwise (as seen from above) in its trip of about 56 feet from the pitcher's hand to the plate. Such a ball will be rotating at a rate of about 1800 rpm, i.e., about one-half the rate of a typical small synchronous electric motor. The side of the ball toward first base (at the top of the figure) then travels about 16 times 9 inches (the circumference of the ball), equal to 12 feet, less than 56 feet, while the side toward third base travels 12 feet farther than 56 feet. The velocity of the third-base side is

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then about 85 mph, while the velocity of the first-base side is only 55. As shown in Figure 2.2, the drag force increases with velocity; i.e., the difference in the pressure of the air on the front face of the moving ball is greater than on the rear face, and that pressure difference increases with velocity. We can then expect the drag force on the third-base side of the ball, which is traveling faster through the air, to be greater than the drag on the first-base side, which is traveling more slowly, and the ball will be deflected toward first base.

While a complete description of the Magnus force would surely be much more complex than the foregoing description, which follows Newton, for a simple and useful picture it seems to me to be relatively transparent and to contain as much of the truth as any other simplistic picture. Note that although the Magnus force probably does vary somewhat with the orientation of the stitches with respect to the spin axis of the ball, the stitches do not contribute to the force in any simple way. Spinning smooth balls, such as those used in table tennis, curve with no aid from stitches.

If the resistive drag force varies only as the square of the velocity and the Magnus force is only an imbalance in that resistance which follows from the faster motion of one side of the ball through the air than the other, we should expect that the Magnus force would be proportional to spin frequency, proportional to the air velocity (or ball velocity), and proportional to the value of the drag coefficient at that velocity.

Though the Magnus force can be described qualitatively in terms of such an imbalance of the drag forces, a reliable quantitative description of the force on a stitched baseball is not available (or more properly, measurements that should be reliable do not agree). Hence, I adopt a model of the Magnus force on a baseball that I believe will reflect, correctly, the general characteristics of the real force as expressed by the drag imbalance—and that, at worst, is not likely to be seriously in error. So from this view the Magnus force, described by the imbalance of resistive forces on the ball that follows from the imbalance of velocity of the airflow past the spinning ball, is proportional to the rate of change of the drag resistance with velocity which is the slope of the drag resistance curve shown as the solid line in Figure 2.2. From this model of the Magnus force, the magnitude of the force (expressed in units of the weight of the ball) is shown in Figure 2.2 as a broken line for balls rotating at a rate of 1800 rpm, which is near the maximum for thrown balls. I estimate that the uncertainties in my estimate of the magnitude of the Magnus force on the baseball, as plotted in Fig. 2.2, are about 25 percent.

Like the related drag force, the Magnus force must depend to some extent on the stitch configuration and thus the orientation of the spin axis. The values plotted in Figure 2.2, then, represent a kind of average over axes orientations.

According to this model, the maximum Magnus force on a ball spinning at a rate of 1800 rpm is seen to be about one-fourth of the weight of the ball. Hence we cannot expect a ball spinning at that rate to curve more than one-fourth the distance it will fall under gravity. Since the variation of velocity is proportional to the rate of spin, the Magnus force on a ball will be proportional to that spin rate; e.g., the force on a ball spinning at a rate of 900 rpm will be one-half that shown by the broken line in the figure.

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*This relation was found in measurements by R. Watts and R. Ferrar and reported in the American Journal of Physics in 1987.*
Note that the Magnus force on a ball spinning at a constant rate increases with velocity up to speeds near 60 mph—the speed of a curveball—and then probably levels off. According to this model of the force, for a given spin rate the Magnus force is no greater for a ball thrown at a velocity of 90 mph—such as a major league fastball—than it is for a 60-mph ball. We might expect, then, that a hard-thrown ball will curve much less than a slower pitch, though the spin rates might be the same. The Magnus force can be expected to be about the same for the fastball, but the force has less time in which to act, as the ball reaches the plate sooner.

The forces that cause the ball to be deflected must also generate a torque that slows down the spin. For a hard-hit ball traveling with an initial velocity of 110 mph, I use the model of the Magnus force I have adopted to estimate that the spin rate will decrease at a rate of about 20 percent per second. For a 400-foot home run, the backspin applied by the bat (perhaps 2000 rpm) would be reduced to about 500 rpm when the ball lands about five seconds later. This is consistent with experience: Fly balls do not spin out of the gloves of outfielders when they barely catch the ball in the tip of their gloves—sometimes called “snow cone” catches—but balls do sometimes spin out of the gloves of catchers fielding foul pop-ups, whose terminal spin rates can be much greater.

**DISTANCE OF FLIGHT OF A BATTED BALL**

Whether it is projected by arm or bat, the distance a ball will travel can be calculated using the simple ballistic relations governing a body in flight and taking the retarding drag force in the direction of motion and the Magnus force, normal to the direction of motion, from the values shown in Figure 2.2. Typical trajectories are shown in Figure 2.4, and a graph of the maximum distance vs. the initial ball velocity is shown in Figure 2.5. From these calculations, the maximum distance is obtained with balls projected at an initial angle of about 35° from the horizontal, though balls projected at 30° or 40° travel almost as far. Note that the ball falls at a rather steep angle at the end of its flight; the trajectories are not symmetric. We find that the 385-foot fly ball hit at the optimum angle of about 35 degrees will be in the air for about five seconds; a really high fly ball—or pop fly—will stay in the air more than six seconds. To put this into perspective, an average right-handed batter will run from home to first in about 4.3 seconds.*

Obviously these trajectories and maximum distances depend upon the drag and the drag coefficients, which are imperfectly known. Assuming an uncertainty of 10 percent in the drag coefficient at high velocities, the uncertainty in the maximum distance of a ball’s flight will be about 3 percent, or about 12 feet for a 400-foot home run. Since we are primarily interested in differences in distance that follow from changes in conditions, such an accuracy is acceptable.

As a ball is thrown or batted for distance in a real situation, it usually has appreciable backspin (on the order of 2000 rpm), which generates a significant force perpendicular to the direction of motion, generally upward. Though the spin will probably increase the drag on the ball slightly, as a consequence of the lift provided by backspin, the Magnus force will somewhat increase the distance traveled by balls hit or thrown at smaller angles and reduce the distance of balls hit or thrown at larger angles. In general, the backspin can be expected to produce a modest increase in the distance the ball will travel, as well as a decrease of a few degrees in the optimum angle of projection. According to my cal-

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*A very fast left-handed batter will reach first base in about 3.7 seconds after a drag bunt, a few tenths of a second—a step—faster than an equally fast right-handed batter, while an unusually slow runner, such as catcher Ernie Lombardi of the 1940s, might take more than 5 seconds to go from home to first. In a timed race like effort in 1921, Maurice Archdeacon circled the bases in 13.4 seconds and the legendary James “Cool Papa” Bell of the Negro Leagues is said to have made it in less than 13 seconds. To put these times into context, a world-class sprinter like Carl Lewis in the 1980s, running with track shoes on a carefully prepared track, would take about 11 seconds to run the 120 yards if the bases were laid out in a line. He would need about 11.6 seconds to run in a straight line the 127 yards that a reasonable course around the bases would measure.
the drag, hence decreasing the flight distance. Since neither effect is known precisely, the magnitude of the backspin effect is uncertain.

Since the backspin itself depends in detail on the exact manner in which the ball is thrown or struck by the bat, to calculate the representative trajectories and distances given in Figures 2.4 and 2.5, I have assumed a backspin of about 1 rotation per 5 feet of flight. Thus the rotation rate is proportional to the velocity. For a ball traveling at 100 mph, that rate will be about 1800 rpm.

Obviously the distance a ball will travel is strongly affected by the wind. Figure 2.5 also shows the effect of winds of 10 mph (near the average velocity throughout the United States) on the flight of a 400-foot home run. With the wind behind the batter, the ball will go about 30 feet farther; with the wind against the batter, the 400-foot home run to center field will end up as 370-foot out.

Although the effects of soft breezes may seem small, the consequences are not necessarily trivial. My analyses of the distribution of fly ball distances and home runs suggest that for an average player, a 1 percent change in the distance the ball travels translates to about a 7 percent change in the probability of hitting a home run. I bring this estimate into our experience by considering that this ratio leads to about one ball being caught on the warning track, or landing in the first row of the stands, for every ten home runs. The ratio is not so large for the great home run hitters: Ruth, Maris, McGwire, and Sosa didn’t hit many balls to the warning track. When they connected, the ball went out.

In general, ballparks are laid out so that the line from “home base through the pitcher’s plate to second base shall run East-Northeast,”* in order that no batter would have to face a Nolan Ryan fastball in the late afternoon with the sun in his eyes. Hence, the prevailing westerlies of the Northern Hemisphere tend to blow out toward right-center field, helping the batter. While for most outdoor ball parks the effect of the wind down on

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*Official Baseball Rules, Section 1.04.
the playing field is reduced as a consequence of the protection afforded by the stands, the long, high home runs can still be carried along by a following breeze.

Since the retarding force on the ball is proportional to the density of the air, the ball will travel farther in parks at a high altitude. A 400-foot drive by Sammy Sosa or Mark McGwire at Shea Stadium, near sea level, on a windless summer day would translate to a 404-foot drive in Atlanta on the Georgia Piedmont, at 1050 feet the highest park in the majors before 1994. The same home run could be expected to go about 403 feet in Kansas City and 403 feet at the Metrodome in Minneapolis or Wrigley Field in Chicago. These differences are not so great as to modify the game, but Sosa could expect his long drive to travel about 420 feet at mile-high Denver. And if the major leagues are further internationalized someday . . . say to Mexico City, at 7,800 feet, Sosa’s blow could sail nearly 430 feet. Figure 2.6 shows the trajectories of long home runs hit with an initial velocity of 110 mph at an angle of 35° at sea level, in Denver, and in Mexico City. Old home run records will be swept away unless the fences are moved out in the high parks.

But even if the fences are adjusted, the high-altitude stadiums will still be a batter’s boon and a pitcher’s bane. With fences moved back, there will be acres of outfield for balls to fall into for base hits, and, though the pitcher’s fastball will be about six inches quicker in Denver, the curve will bite about 20 percent less, which is more important.

With less drag, the ball will also get to the outfielders faster in Denver than at Fenway Park in Boston. Players for the Colorado Rockies have noted that in Denver’s outfield, “Fly balls come at you faster and sail farther than you might expect.” Indeed, a hard-hit “gapper” between the outfielders will reach the 300-foot mark about two-tenths of a second faster in Denver than at sea level, cutting down the pursuit range of an outfielder by five or six feet—not insignificant in this game of inches. Even the range of a shortstop covering a line drive or one-hopper will be cut by about a foot in Denver.

The use of a less lively, “high-altitude” ball would reduce the altitude effect, just as special less lively, “high altitude” balls are used in tennis, though for somewhat different reasons.

Temperature, barometric pressure, and humidity also affect the flight of a ball. The canonical 400-foot home run will go about three feet farther for every one-inch reduction in the barometer and as much as ten feet farther on a 95° July day in Milwaukee than on a 45° April day. The effect of temperature differences on the elasticity of the ball will also have an effect on the distance a batted ball travels.

Humidity per se has little effect on the flight of the ball. Indeed, since water vapor is a little lighter than air, if all other factors are the same, a ball will travel farther if it is exceptionally humid, though only by a few inches. The general belief that balls do not travel as far if the humidity is high probably stems from people’s experience of windless humid nights when the temperature has dropped from its daytime highs. Then, with the cooler evening air a little denser and no breeze to carry the ball, the ball that would have cleared the fence for a home run in the hot afternoon carries only to the warning track.

Humidity, however, certainly affects the weight and elasticity of balls in storage. Balls stored under conditions of high humidity will gain some weight through the absorption of water from the
air, and their elasticity (the coefficient of restitution, discussed in Chapter 5) will be reduced.

Since the retarding force is proportional to the cross-sectional area of the ball, small balls will go farther than large balls. According to the Official Baseball Rules, a ball may be as large as 9½ inches in circumference and as small as 9 inches, as light as 5 ounces and as heavy as 5½ ounces. The stroke that propels a larger ball 400 feet will drive a smaller ball—of the same weight and elasticity—perhaps six feet farther. The stroke that drives the lighter ball 400 feet will drive the heavier ball—of the same size and elasticity—the same distance, give or take a foot or two. Though the heavier ball will come off the bat a little slower, its greater sectional density will carry it better through the drag of the air and just about compensate for the deficit in initial velocity.

TECHNICAL NOTES

a) The Reynolds number, Re, is the ratio of inertial and viscous forces. The value of the number for a sphere of diameter, r, moving with a velocity, V, through a fluid of density, ρ, and viscosity, μ, is:

\[ Re = \frac{\rho V r}{\mu} \]

For a baseball, Re = 2200 V, where the V is measured in mph.

For a given ball velocity, Re is smaller at altitude even as the density is reduced. The value of Re is also reduced with increasing temperature, as that change decreases the air density, which is proportional to the absolute temperature (the temperature above absolute zero, about -460°F). Such an increase in temperature also reduces Re by increasing the viscosity of the air, μ, which is proportional to the absolute temperature of the air—though independent of the air density.

b) The relation for the drag force, \( F_d \), can be written as:

\[ F_d = C_d \rho V^2 \frac{r^2}{2} \]

Here A = \( \pi r^2 \), with r the radius of the sphere, is the cross-sectional area of the sphere, \( \rho \) is the density of the air, V is the velocity of the ball, and \( C_d \) is the drag coefficient. For \( C_d = 2 \), this is just the force required to move a column of air the size of the ball to match the velocity of the ball.

If the air passing by an object is divided into small packets—with each perhaps marked by a smoke particle—and the packets flow about the object in paths marked by smooth lines that do not change their position relative to the ball over time as the flow proceeds, we call the paths “stream-lines.” If the air were to flow in completely smooth stream-lines around the ball, generating neither trailing vortices nor turbulence (which do change over time), after the ball has passed, the elements of air in back of it would be just as they were before it passed. Hence no energy would have been transferred to the fluid and the drag would be zero—and thus the adjective “streamlined” to describe airplane and automobile shapes that reduce aerodynamic drag. But a baseball is not very well streamlined.

c) For velocities such that the drag coefficient, \( C_d \), does not vary strongly with the velocity, V, of the ball through the air, the Magnus force, \( F_m \), can be expressed as:

\[ F_m = K f V C_d \]

where the force, \( F_m \), is measured in pounds-force, the ball velocity, V, is expressed in mph, and the spin frequency, f, is measured in rpm. The measurements of Watts and Ferrar suggest that \( K \approx 2 \cdot 10^{-6} \).

Almost all of fluid dynamics follows from a differential equation called the Navier-Stokes equation. But this general equation has not, in practice, led to solutions of real problems of any complexity. In this sense, the curve of a baseball is not understood;
the Navier-Stokes equation applied to a baseball has not been solved. Professor Robert Romer, long-time editor of the American Journal of Physics, told me of an eminent physicist who said, "There are two unsolved problems which interest me deeply. The first is the unified field theory [which describes the basic structure and formation of the universe]; the second is why does a baseball curve? I believe that, in my lifetime, we may solve the first, but I despair of the second."

Therefore, the foregoing equation used to describe the Magnus force follows from the fundamental Navier-Stokes equation only after that equation is simplified through some rather drastic approximations. My simple Newtonian description of the complex processes that govern the curveball does not contain all of the truth. But it is useful, and surely reasonably accurate for baseball velocities under 75 mph. In the absence of good measurements at higher velocities, we can be less certain of my estimate of the Magnus forces for balls traveling at greater speeds. In particular, the lack of increase with velocity in the drag force at baseball velocities near 90 mph has not been demonstrated experimentally, and the variations of the drag and Magnus forces with the orientation of the axis of the ball, important in this game, are not well known.

Incidentally, the Magnus effect responsible for the curve of the curveball and the hop of the fastball is not quite the same as the Bernoulli effect; it is, indeed, more than the Bernoulli effect—which is why it is called the Magnus effect and not the Bernoulli effect.

d) To consider the Magnus force at velocities greater than 75 mph, we must take into account the variation of the drag coefficient. In general, that coefficient, \( C_d \), varies strongly with velocity near the transition between smooth and turbulent flow, and we might reasonably expect the Magnus force, \( F_m \), to vary as:

\[
F_m = k f v C_d [1 + 0.5 \cdot \left( \frac{dC_d}{dv/V} \right)]
\]

This differential form follows naturally from the basic equations for both the low-velocity smooth flow and the high-velocity turbulent flow. According to this relation, the Magnus force can reverse sign when the logarithmic differential \( (dC_d/C_d)/(dV/V) \) is less than -2, which can occur at the drag crisis when the drag force decreases sharply as velocity increases. This description is not soundly based on theory, but it accounts for the reversed Magnus force seen for smooth balls.

The failure of the Magnus force to increase with velocity for baseballs at velocities from 70 to 100 mph that follows from these arguments, and from the foregoing formula (as shown in Figure 2.2), is the same effect that reverses the sign of the force for balls with more uniform surfaces. This reversal was observed by Briggs in wind tunnel measurements of the Magnus force on smooth spinning balls the size of a baseball. For golf balls with the dimples smoothed out, that dip is sufficiently great as to reverse the Magnus effect, so that a ball hit with backspin ducks into the ground rather than rising, as do normally dimpled balls.

e) Even as the interaction of the spinning ball with the airstream causes the ball to be deflected, there must be a reaction on the ball that reduces its spin rate. The unbalanced drag forces shown in Figure 2.3 that generate the Magnus effect must also produce a countervailing torque, slowing down the spin.

In the wind tunnel system of Figure 2.3, the interaction of the surface of the spinning ball with the air tends to slow down the air more on the side of the ball moving faster through the air (the third-base side in Figure 2.3) and less on the other side (the first-base side in the figure). This differential change in the velocity of the air results in a countervailing change in the pressure through the Bernoulli effect—the faster the flow, the lower the pressure—which then contributes in a major way to the Magnus force. But those changes in the velocity—and thus momentum—of the air that differ on the two sides of the ball require a tangential force that must be supplied by the moving ball surface.

Hence, we can expect a reactive torque, \( L \), that resists the spin, such that:

\[
L = k F_m r
\]
where \( F_m \) is the Magnus force, \( r \) is the radius of the ball, and \( k \) is a proportionality constant that I estimate as \( \approx 1/10 \). (I note that the estimate is not solidly based.) The energy-per-second or power, \( P \), lost to the spin kinetic energy, \( T \), is then:

\[
P = L \omega = kF_m r \omega
\]

where \( \omega = 2\pi f \) is the angular spin velocity of the ball.

The spin-down time constant, \( \tau \), is then:

\[
\tau = \frac{T}{P} \quad \text{where} \quad T = \frac{1}{2} mr^2 \omega^2
\]

where \( m \) is the mass of the ball.

The time constant, \( \tau \), is a measure of the time required for the ball to lose its spin. Thus, for, \( \tau = 5 \) seconds, a value that I estimate to be typical of normally spinning fly balls, the spin rate will decrease by a factor very near \( 1/\tau = 1/5 \) for each second the ball is in flight.

At high spin rates, the drag force on a baseball may increase. Some wind tunnel measurements show substantial increases in drag for spinning balls; some do not. In science, unlike many other disciplines, if two results differ widely, the truth is more likely to be found at one of the extremes than in the middle. However, my experience with the game of baseball itself leads me to believe that the drag on a baseball does not increase strongly with spin.

For moderate spins, where \( v'/v < 1/4 \), I assume that the drag is increased by a factor of \( 1 + (v'/v)^2 \), where \( v' = \omega r \) is the rotational velocity of the surface of the ball and \( v \) is the ball velocity. This form fits some measurements and has a theoretical basis. For larger values of \( v'/v \) I use a more arbitrary recipe, but that only affects pop flies and foul balls, where there are other uncertainties as well.

\( f) \) For baseballs moving at very high velocities—over 120 mph—or at low velocities—less than 60 mph—the drag coeffi-

cient, \( C_p \), does not vary much with velocity. At these velocities
the drag resistance on a baseball is simply proportional to the
density of the air and hence, for a given temperature, to the barometric
pressure. An increase in altitude of about 275 feet (equal
to a reduction in pressure of a third of an inch of mercury on the
barometer) or an increase in temperature of about 5° F decreases
the drag by about 1 percent.

But the situation is more complicated for the intermediate
velocities that are most important to baseball. At these velocities,
where the transition is taking place from high-drag smooth flow
to low-drag turbulent flow conditions, the drag coefficient is
changing from values of about 0.5 at low velocities to values of
about 0.25 at high velocities. Since the drag coefficient is known
to be largely dependent on the Reynolds number, which depends
on the pressure and temperature as well as velocity, the "position"
of the transition as seen on Figure 2.1 moves to higher
velocities as temperature and altitude increase. Thus this effect,
acting alone, increases the drag on a baseball moving at the ordi-

nary velocities of the sport as either the temperature or altitude
increases, partially countering the reduction in drag caused by
the decreased density of air being pushed aside by the ball.

We do not know the precise variation of the drag coefficient
with velocity well enough to reliably calculate the effects of that
variation on the drag force, but I estimate (that is, I make an ed-
guessed guess) that the variation in the transition with temperature
and altitude reduces the density effects by about a factor of two.
With this factor included, the drag is reduced by 1 percent for
each increase in altitude of 550 feet and for each increase in tem-
perature of 10° F.

For the 400-foot home run, this 1 percent decrease in drag by
1 will add a little less than two feet to the ball's carry and increase
the probability of the average player's hitting a home run by
roughly 3.5 percent.

When all is considered I find that, as a consequence of the
reduced air density at high altitudes, the drive projected at an
angle of 35° upward at sea level that lands 400 feet from home plate will travel an extra 4 feet for every thousand feet of altitude where conditions are otherwise the same. A 350-foot drive will gain an extra 4.6 feet per thousand feet of altitude. The approximation that the increase in distance is proportional to the increase in altitude is probably good to a few percent.

The problem of the smooth-turbulent transition variation does not occur for golf balls, where the dimples place the ball in the turbulent region for most velocities important in golf. Thus the drag on a golf ball is simply proportional to the air density. With that simplification, I find that a golf ball will fly about 11 percent farther in Denver than at sea level, a conclusion confirmed by professional golfers.

g) R. C. Larsen found that the weight of balls stored at 100 percent humidity for four weeks increased by 11 percent and that the coefficient of restitution at an impact velocity of 25 mph decreased by 10 percent—when dropped on concrete from a height of 20 feet, the humidified balls will bounce only about 80 percent as high as the balls stored at low humidity. If that proportional decrease in elasticity would hold at greater-impact velocities, the swing of the bat that would drive a "dry" ball 380 feet would propel the ball stored at high humidity only 350 feet. After sitting in dry air for a few hours to dry their covers, the humidified balls could not be distinguished from normal balls by a layman. However, an experienced pitcher would have probably noticed that they were heavier and softer than the balls he was used to.

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THE SWING OF THE BAT

A MODEL OF BATTING

There are many ways of batting the ball successfully through swinging at the ball in a manner such that precision of placement of the batted ball is important rather than high velocity. The drag bunt and the hit-and-run behind the runner at first base, for instance, represent purposeful hits that are made by swinging with less than full power. Hits to the opposite field often fall into the same category. Also, the batter may be fooled by the pitcher to such a degree that his timing is thrown off and he swings weakly—but sometimes successfully, hitting, perhaps, a "Texas leaguer" or an infield grounder which bounds so slowly that he beats out the throw to first. In the following discussion, I don't consider the imperfect swings—though they may constitute the majority—but only the full swing made with maximum effort. Just a few players—Babe Ruth and Ted Williams come to mind—always seemed to make full swings.

In considering the mechanics of the batting process, it is useful to construct a model of a full swing that is tractable but sufficiently close to a real swing by a real batter that the consequences