Our current understanding of the laws of nature is based on the concept of forces. We know of four basic forces (Lect. 37): Gravity, electromagnetic, weak, and strong force.

**Gravity** is unique, because it applies to everything.

The other forces are more selective. For example, electromagnetism applies only to charged objects.
Newton’s gravity

From Newton’s Principia, 1615

Throwing a ball fast enough puts it into orbit. The same argument explains why the Moon orbits the Earth. This was the revelation that came to Newton under the apple tree.
Gravitational force

Gravitational force on the apple by the Earth
Gravitational force on the Earth by the apple

These forces are equal and opposite.

The Earth and the apple play the same role, despite their large mass difference.
Newton’s law of gravity

\[ F = G \cdot \frac{m_1 \cdot m_2}{d^2} \]

- The gravitational force \( F \) between two objects increases with their masses \( m_1, m_2 \) and decreases like \( 1/d^2 \) with their distance \( d \).
- The pre-factor \( G \) is the gravitational constant.
- The gravitational acceleration is \( g \) (Slides 10-13).
Gravitational force decreases with distance from Earth

Force on apple \( F_{\text{apple}} = 6.7 \times 10^{-11} \frac{m_{\text{Earth}} \times m_{\text{apple}}}{d^2} \) (in kg and m)

So moving farther from the Earth should reduce the force of gravity.

An airplane cruises at about 8 km altitude.
\( \rightarrow d \) increases from 6,370 km to 6,378 km.
\( \rightarrow \) only a 0.25% change in gravity
The international space station orbits at 350 km.

→ $d = 6,370 \text{ km} + 350 \text{ km} = 6,720 \text{ km}$

→ Again $d$ has changed only a little, leading to a 10% decrease in gravity.
So why is everyone floating around?

Edward M. (Mike) Fincke, Expedition 9 NASA ISS science officer and flight engineer, is pictured near fresh fruit floating freely in the Zvezda Service Module of the International Space Station. (NASA)

James S. Voss, Expedition 2 flight engineer, looks over an atlas in the Zvezda Service Module. (NASA)
The space station is falling ...

... similar to Newton’s apple and the Moon

- In its circular orbit the space station is continuously falling towards the Earth but never reaching it.

- Everything inside is also falling towards the Earth, giving astronauts the impression of being weightless.

- In Lecture 16, this situation will be explored further to develop Einstein’s generalization of Newton’s gravity.
Path of free-falling object, thrown upwards.
Newton’s other famous law:

\[ F = m \cdot a \]

A force \( F \) causes an acceleration \( a \) of an object with mass \( m \).

- What is **acceleration**?
  Change of velocity \( v \) with time \( t \):
  \[ a = \frac{\Delta v}{\Delta t} \quad \left[ \frac{\text{m}}{\text{s}^2} \right] \]

- What is **velocity**?
  Change of position \( x \) with time \( t \):
  \[ v = \frac{\Delta x}{\Delta t} \quad \left[ \frac{\text{m}}{\text{s}} \right] \]

(\( \Delta \) designates a small interval.)
Position \( x(t) \) plotted versus time \( t \)

Velocity \( v(t) = \text{slope of } x(t) \)

Acceleration \( a(t) = \text{curvature of } x(t) \)

Example: Constant acceleration during free fall.
Supreme Scream:  
Fall 300 feet in 4 seconds

The acceleration due to the force of gravity: \( g \approx 10 \text{ m/s}^2 \)

Each second the velocity \( v \) increases by 10 m/s.

After 1 second:  
\[ v \approx 10 \text{ m/s} = 22 \text{ miles/h} \]

After 2 seconds:  
\[ v \approx 20 \text{ m/s} = 44 \text{ miles/h} \]

After 3 seconds:  
\[ v \approx 30 \text{ m/s} = 66 \text{ miles/h} \]
Acceleration parallel to the velocity: 
Free fall

Use both laws to calculate the acceleration $g$:

$$g = \frac{F}{m} = \frac{G \cdot m_{\text{Earth}}}{d^2} \approx 10 \text{ m/s}^2$$

$d = \text{radius of the Earth}$
Acceleration perpendicular to the velocity:
The Moon

Newton:
The moon falls towards Earth, like the apple.
Derivatives in physics and in the stock market

The velocity $v(t) = \frac{\Delta x}{\Delta t}$ is called the derivative of $x(t)$. Likewise one can define the derivative $\frac{\Delta P}{\Delta t}$ of a stock price $P(t)$ which describes its change over time.

Stock traders bet on the derivative of a stock price by “selling short” or “buying on margin”. A pessimist bets that the stock price $P$ will go down after a certain time $\Delta t$ and “sells short”. An optimist bets that a stock will go up and “buys on margin”. If the stock goes up, the pessimist pays the optimist the price change $\Delta P$. If it goes down, the optimist pays the pessimist. Neither of them actually owns the stock. They make big bets with little cash reserves – a dangerous game.
Extrapolation via derivatives

A smooth curve can be extrapolated from the past into the future using its derivatives:

If $P$ is a stock price:

1. **1st derivative** says “buy”
2. **2nd derivative** says “not so fast”

Since the stock price is not a smooth curve, sophisticated statistical methods are needed for an extrapolation. This is done by quantitative analysts (“quants”) using computer trading programs - a lucrative business for some.
Is the stock market predictable?

Economists are working on theories to answer this question. One theory goes like this: The stock market is completely unpredictable, because otherwise some quant could write a computer trading program that extracts money from the stock market. But there are quants who make billions of dollars doing just that.