Maxwell’s equations: Case studies

1) Charges cause electric fields.
2) Currents cause magnetic fields.
3) Changing electric fields cause magnetic fields.
4) Changing magnetic fields cause electric fields.

E = Electric field
\( \rho \) = Charge density
B = Magnetic field
\( J \) = Current Density
\( \nabla \) = Derivative in space
\( \nabla \times \) = Cross derivative
\( \mu_0 \varepsilon_0 = \frac{1}{c^2} \)
\( \nabla \cdot E = \frac{\rho}{\varepsilon_0} \)
\( \nabla \cdot B = 0 \)
\( \nabla \times E = -\frac{\partial B}{\partial t} \)
\( \nabla \times B = \mu_0 \varepsilon_0 \frac{\partial E}{\partial t} + \mu_0 J \)
\( \nabla / \partial t = \) Derivative in time ("changing")
Maxwell’s Equation #1
Charges cause electric fields

Sparks and Lightning

Separate enough charge, and the electric field causes forces strong enough to tear electrons from the molecules in air (electric discharge).
Maxwell’s Equation #2

Currents cause magnetic fields.

Iron filings align with magnetic field lines

A magnetic field loops around the current flowing in a wire.
The Earth’s magnetic field

The magnetic field is generated by currents flowing in liquid metal deep inside the Earth.

Opposite magnetic poles attract each other. The magnetic North Pole of a compass (in red) is attracted to the magnetic South Pole of the Earth, which lies close to the geographic North Pole.
Geomagnetism

- The magnetic field of the Earth has reversed its direction many times, typically every few hundred thousand years.
- One knows that from the magnetization of iron-rich lava. It gets magnetized by the Earth’s magnetic field during cooling and keeps its magnetization after it solidifies.

Time (in millions of years)

Black and white are for opposite magnetic polarity.
Maxwell’s Equation #3
Changing electric fields cause magnetic fields.

An electromagnetic wave is created by the changing electric field of a spark, an antenna, or an oscillating molecule (greenhouse gas). The changing electric field then creates a changing magnetic field.

Electromagnetic wave
Maxwell’s Equation #4

Changing magnetic fields cause electric fields.

Electric generator

The magnetic field across the wire loop changes as the loop rotates. That causes an electric field, which then generates a current around the loop.
Electric generator of a power plant, driven by a steam turbine
Electric Motor

An electric motor can be viewed simply as an **inverted electric generator**. The current is the input (rather than the output), and the rotary motion is the output (rather than the input).

As a result, it is possible to switch an electric motor from consuming electric power to producing power.

This process is called **regenerative braking**. It converts kinetic energy to electric energy and thereby provides **additional energy efficiency** to electric cars and trains.