The force law in relativity

- Consider an object that is accelerated by a constant force $F$, for example a space traveler in a rocket ship.

- Newton’s force law $F = ma$ predicts constant acceleration, which is given by $a = v/t$.

- Consequently the velocity $v = a \cdot t = F/m \cdot t$ continues to increase over time without limits. It reaches the velocity of light $c$ after the time $t = v/a = c/a = mc/F$.

- In relativity, the mass increases as the velocity increases (Lect. 14, Slide 6). Combining Newton and relativity, one would expect the acceleration $a = F/m$ to decrease. This is also true when using the relativistic force law.

- As $v$ approaches $c$, the mass $m$ becomes infinite. And the time $t = mc/F$ to reach $c$ becomes infinite.
An object accelerated by a constant force

- At small velocity the motion is described by Newton’s \( F=ma \).
- Big deviations occur at velocities near \( c \).
- In relativity the velocity never reaches \( c \).
**Time travel**

- A time traveler tries to use time dilation to age less during a long space flight. That requires high speeds.
- **With constant force**, the acceleration weakens at high speed because of the mass increase. This occurs right at the onset of time dilation, since mass increase and time dilation are on the same curve (Lect. 14, Slide 6).
- **Constant acceleration** produces high speeds quickly, but it requires an increasing force to compensate for the increasing mass. The force quickly becomes lethal (Lect. 17, Slide 5).
Relativistic speed limit

- Energy (or mass or information) cannot move faster than the speed of light $c$.

- If one tries to accelerate an object towards the speed of light, its mass becomes infinite. To reach $c$ would require an infinite force.

- Particles with zero rest mass $m_0$ are special (e.g. the photon). They move exactly with the velocity of light, but not faster. They cannot sit still either, since their energy would go to zero ($E=m_0c^2$).
The relativistic version of $F=ma$

Newton’s $F=ma$ is replaced in relativity by a relation between force $F$ and momentum $p$:

$$F = \frac{\Delta p}{\Delta t}$$

The relativistic momentum is: $$p = m v$$

$v$ = velocity

$m$ = relativistic mass (increases with velocity)
Newton’s force law and its relativistic extension can be written in similar form:

Newton: \[ F = \frac{\Delta (m_0 \cdot v)}{\Delta t} \]

Einstein: \[ F = \frac{\Delta (m \cdot v)}{\Delta t} \]

\( m_0 \) is the rest mass, \( m \) the relativistic mass.
Momentum replaces velocity

- The momentum $p = mv$ replaces the rest mass $m_0$ and the velocity $v$ in the relativistic generalization of Newton’s force law $F = \Delta p/\Delta t$.

- Momentum still adds up in relativity, but velocity doesn’t.

- Therefore, momentum is a fundamental variable in physics.

- **Momentum is the counterpart to distance.** Compare reciprocal space (= momentum space) versus real space in diffraction. Momentum is also the counterpart to distance in quantum physics, for example in the uncertainty relation.
Energy and momentum vs. time and space

- Energy and momentum are related to time and space via conservation laws.
- Energy conservation follows from translation symmetry in time.
- Momentum conservation follows from translation symmetry in space.
Space-time diagrams

• In a space-time diagram the position $x$ of a body is plotted horizontally, and the time $t$ vertically.

• Use $c \cdot t$ instead of time to have both coordinates in meters.

• Minkowski developed space-time diagrams to describe motion in relativity.
The world line of a particle

The motion of a particle becomes a line in space-time, the *world line* of the particle.

- World lines appear in Feynman diagrams of particle interactions (Lect. 33).
- They become tubes or sheets in string theory (Lect. 39).
World line of a particle moving with velocity $v$:

$x = v \cdot t \quad c t = c/v \cdot x$

The slope of the line is $c/v$. The smallest slope is $c/c = 1$. 

Space-time coordinates

- Low velocity
- Higher velocity
- Light velocity ($45^0$, $x=ct$)
- Forbidden ($>45^0$, $x>ct$)
Causality in space-time

Space-time consists of three regions separated by the light lines.

- An observer (white dot at the center) cannot communicate with the space-like region (gray), because that would require a signal faster than light. This is the situation for galaxies outside the horizon (Lect. 3, Slide 3).

- Signals can be received from the past (green) or sent into the future (blue). These are the time-like regions.