The space-time 'distance' Δs

- There is a relativistic generalization of the distance Δx (in space) and Δt (in time).

  This is the space-time distance Δs :

  $\Delta s = \sqrt{(\Delta x)^2 - (c\,\Delta t)^2}$ for $\Delta x > c\,\Delta t$ (space-like)

  $\Delta s = \sqrt{(c\,\Delta t)^2 - (\Delta x)^2}$ for $\Delta x < c\,\Delta t$ (time-like)

  $\Delta s = 0$ for $\Delta x = c\,\Delta t$ (light-like)

- The combination of Δx and Δt is chosen such that length contraction compensates time dilation.

- The distance Δs is independent of the observer, like the velocity of light.

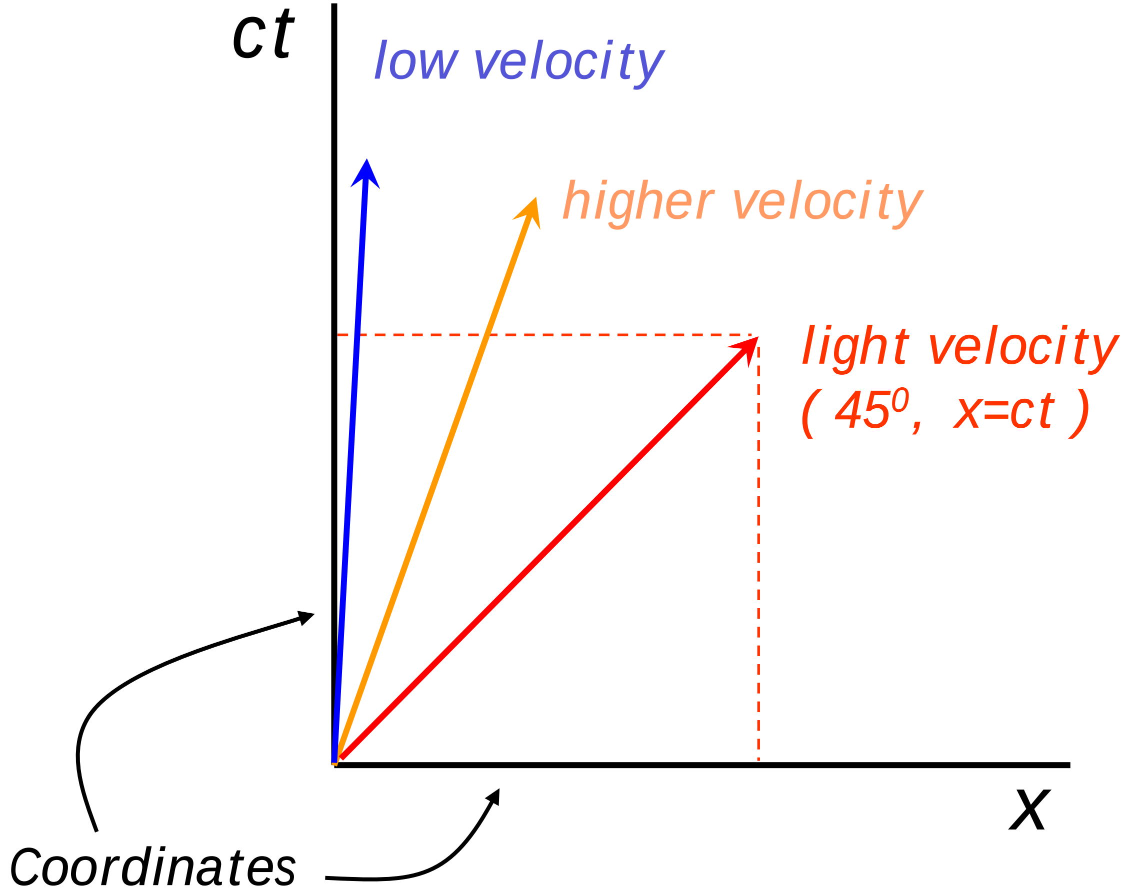
Space-time diagrams

- Minkowski developed space-time diagrams: A geometrical way to understand relativistic motion.
- Think of an event, such as a lightning strike, in terms of position *x* and time *t* .
- Use (x, ct) as 'coordinates' (both in meters).

Constant velocity motion

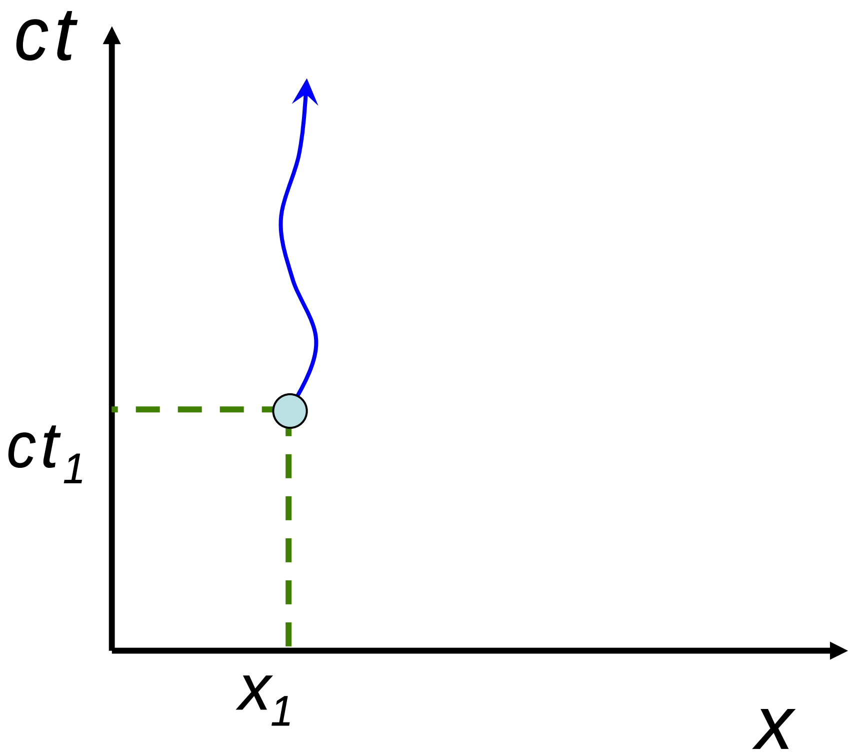
World line of an object moving at constant velocity:

$x = v \cdot t$



The world line of a particle

The motion of a particle becomes a line in space-time, the world line of the particle.

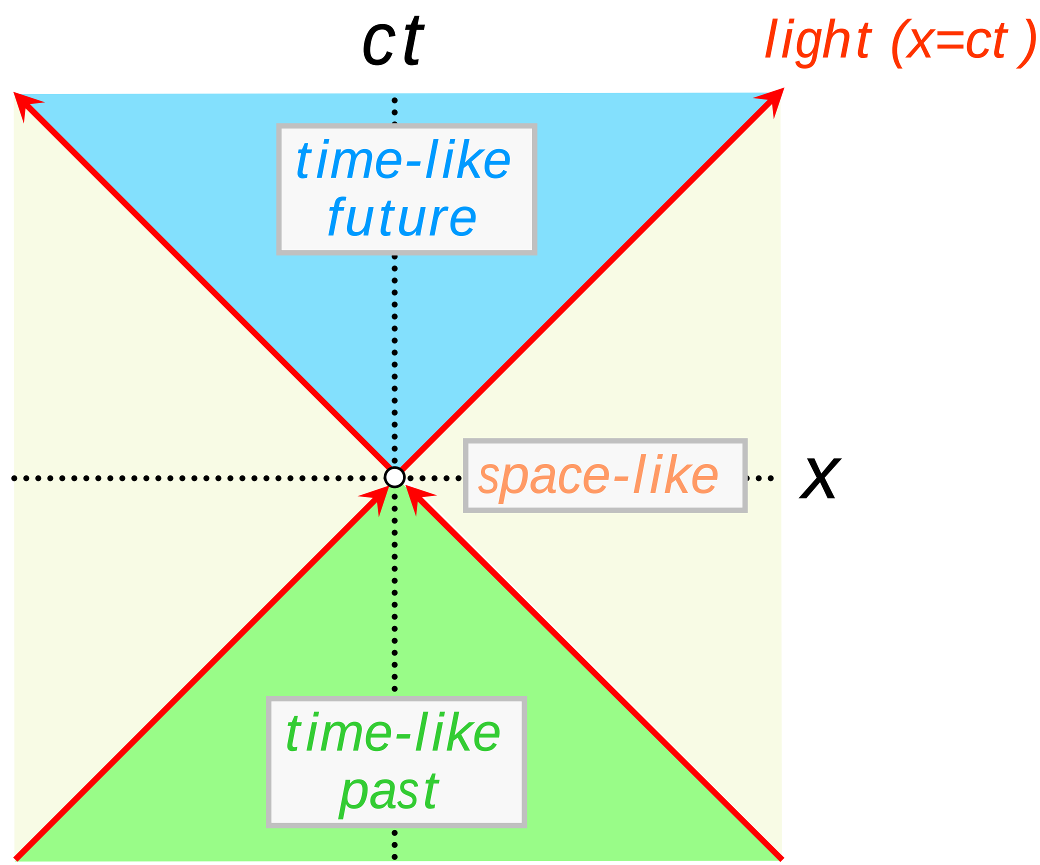


- World lines appear in Feynman diagrams of particle reactions.
- They become pipes or sheets in string theory.

Space-time and causality

Space-time consists of three regions
separated by light lines (red).

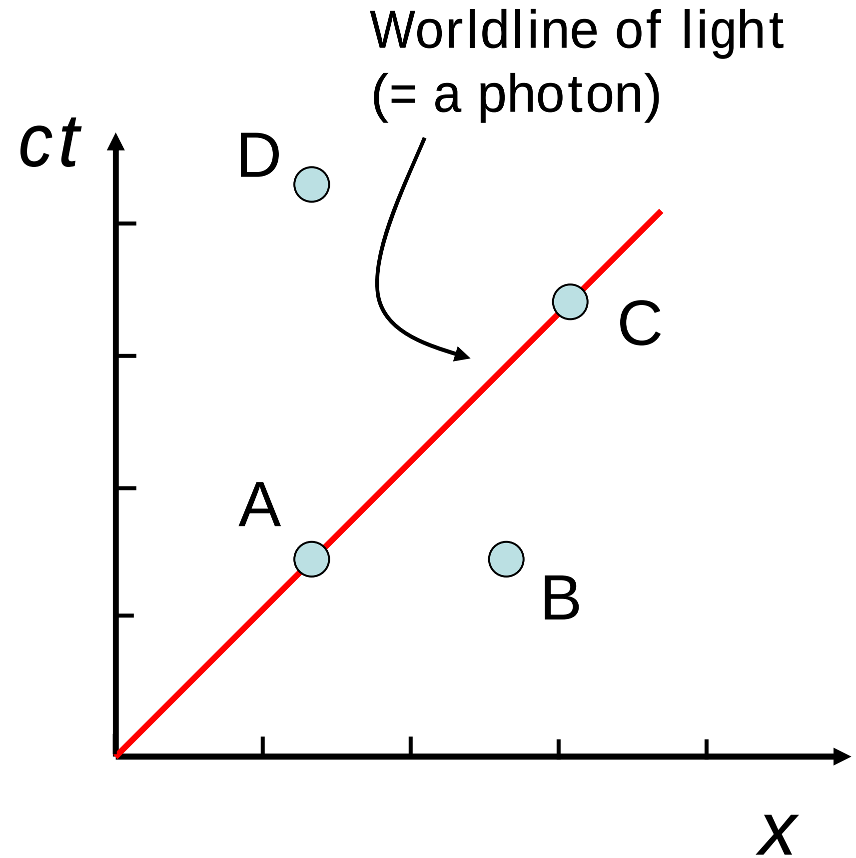
An observer (white dot at the center) cannot communicate with other observers in the space-like region, because that would require signals faster than light. Signals can be received from the past or sent into the future.



Question

Which events have zero space-time separation Δs ?

1. A and B
2. A and C
3. C and D
4. None of them



Forces and energy in relativity:
What about Newton's laws?

- Relativity dramatically alters our perspective of space and time
  - But clearly objects still move,
    spaceships are accelerated by thrust,
    work is done,
    energy is converted.

- How do these things work in relativity?

Applying a constant force

- Particle initially at rest (t=0), then accelerated by a constant force F.

Newton predicts: $F = m a \Rightarrow a = F/m$

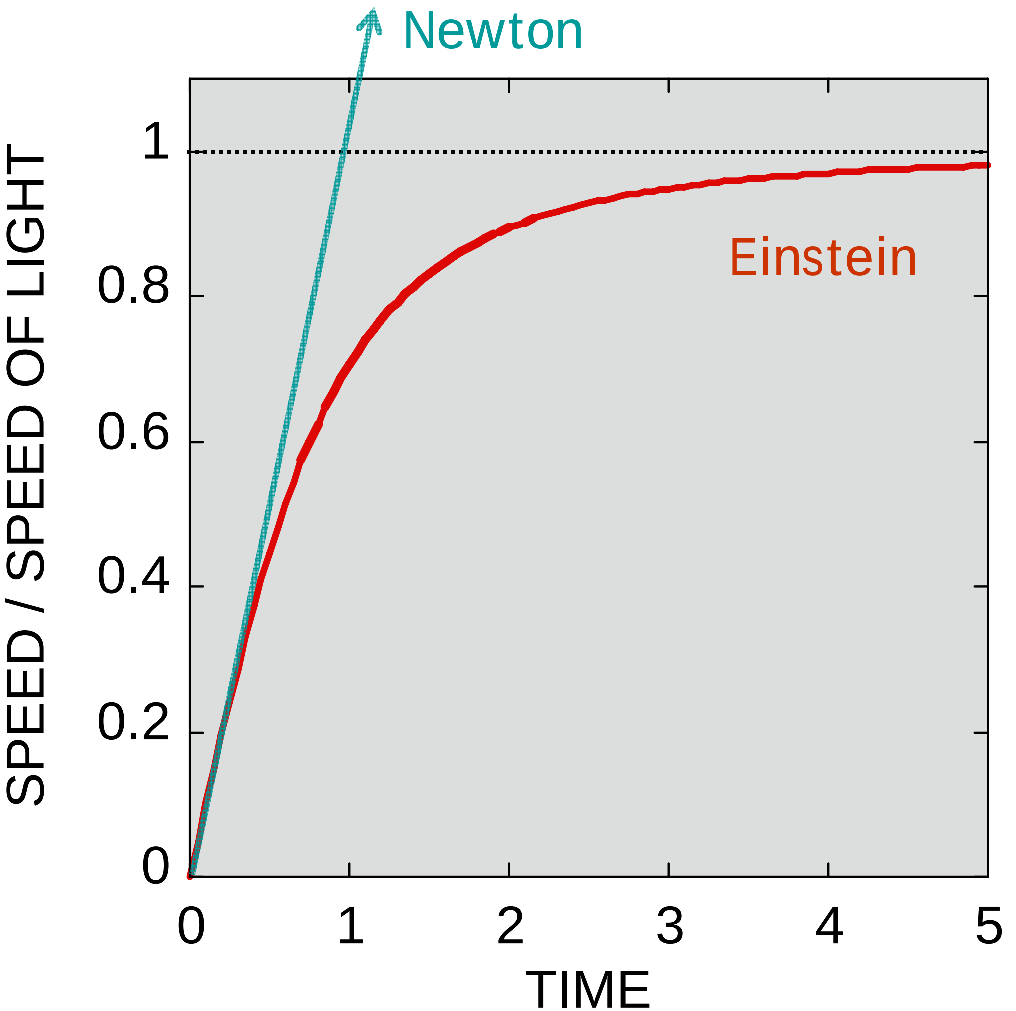
$$velocity = acceleration \cdot time$$
$$= (force/mass) \cdot time$$

- The velocity increases forever as time increases.

Relativity says *no*. Cannot exceed *c*.
The effect of the force must get smaller as the velocity approaches *c*, the speed of light.

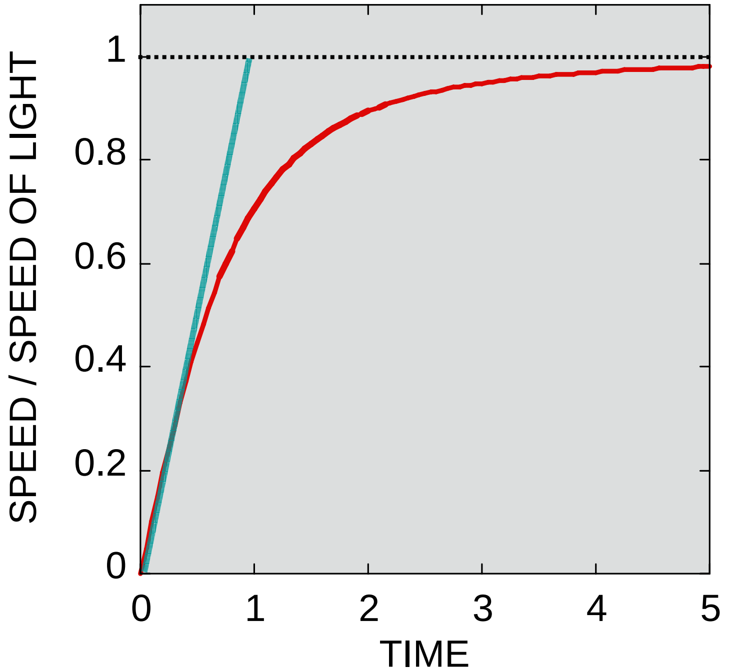
Relativistic velocity of a particle accelerated by a constant force

- At small velocity the motion is described by Newton's $F=ma$.
- At higher velocities, big deviations!
- The velocity never reaches the speed of light in relativity.



Time traveler revisited

- A time traveler tries to avoid the lethal forces of constant acceleration by using constant force instead.
- His acceleration decreases just as time dilation begins.
- This is due to the onset of the relativistic mass increase. A larger mass is accelerated less by a given force.



The relativistic version of $F = m\,a$

Newton's law $F = m\,a$ is replaced in relativity by a relationship between force and momentum:

$$F = \frac{\Delta p}{\Delta t}$$

The relativistic momentum is: $p = m\,v$

v = velocity

m = relativistic mass (see Fig.10.13)

Similiarity between the force laws

Newton’s force law and its relativistic analog can be written in similar form:

Newton: $$F = m_0 \cdot \frac{\Delta v}{\Delta t}$$

Einstein: $$F = \frac{\Delta (m \cdot v)}{\Delta t}$$

m_0 is the rest mass, m the relativistic mass (Fig. 10.13).

Relativistic speed limit

- Energy (or mass) can *never* move *faster* than the speed of light *c*.

- If one tries to accelerate an object (such as an electron) towards the speed of light, its mass and momentum become infinitely large. To accelerate it beyond *c* requires an infinite force $F = dp/dt$.

- Particles with zero rest mass (such as the photon) move at the velocity of light, but not faster. They cannot sit still either, since their energy would be zero.