Conductivity, Mobility, and Carrier Density

\[ J = \sigma \cdot E \]
\[ v = \mu \cdot E \]
\[ \sigma = (n e^2 \tau) / m^* \]
\[ \mu = (e \tau) / m^* \]
\[ J = \frac{n q \cdot v}{v = (q \tau / m^*) \cdot E} \]

defines conductivity \( \sigma \)
(\( J = \) current density, \( E = \) electric field)

defines mobility \( \mu \)
(\( v = \) average velocity = drift velocity)

\( n = \) carrier density \((n_h, n_e)\)
(\( q = \) carrier charge \(\pm e\))

\( \tau = \) scattering time
(\( m^* = \) effective mass)

Carrier Density from the Hall Effect

\[ \frac{E_y}{J_x B_z} = \frac{1}{n q} = R_H \]

\[ \Rightarrow \text{carrier density } n \text{ (including sign); } V_H = \Delta y \cdot E_y \text{ = Hall voltage} \]

Electric and magnetic force on the carriers:

\[ F_{\text{Lorentz}} = q \cdot (E + v \times B) \]

(\( E = \) electric field, \( B = \) magnetic field, \( v = \) velocity)

(\( E = \) Voltage/distance \([\text{V/m}]\), \( B \) in Tesla \([\text{T}]\), \( J = \) current density = current/area = \(\text{A/m}^2\) )

When the current starts, the magnetic part of the Lorentz force deflects electrons/holes perpendicular to their velocity \( v_x \). The charge buildup causes a voltage \( V_y \) across the sample. The electric field \( E_y \) creates a Lorentz force opposite to the magnetic force. Eventually, an equilibrium is established between the magnetic and electric force, and the electrons/holes move along a straight line: \( 0 = F_{\text{Lorentz}} = q \cdot (E_y + v_x B_z) \)

Together with \( J_x = n q \cdot v_x \) one obtains the formula for the carrier density \( n \) given above. Electrons and holes generate opposite electric fields, such that one can tell the doping from the sign of the Hall voltage (p- versus n-type; one of them always dominates).
Semiconductor Junctions

These diagrams are for the potential energy \( U = -e \cdot V \) of an electron. The electrostatic potential \( V \) has the opposite sign because the electron has negative charge.

Start out with two neutral pieces. Put them into contact and let electrons flow into lower energy levels until the two Fermi are equilibrated. Neutral dopants become ionized and build up an electrostatic potential (“space charge”). For simplicity, the bulk Fermi level is placed between the dopant level and the band edge (true at low temperature).

1a) Semiconductor – Semiconductor: **pn – Junction**

Separated p- and n-type semiconductors:

\[
\begin{align*}
\text{n-type} & \quad \text{p-type} \\
\begin{array}{c}
\text{CB} \\
\text{E}_F \\
\text{VB}
\end{array} & \quad \begin{array}{c}
\text{CB} \\
\text{E}_F \\
\text{VB}
\end{array}
\end{align*}
\]

After forming the pn-junction:

Electrons flow downhill from n to p until the Fermi levels \( E_F \) line up.

1b) Semiconductor – Semiconductor: **Heterojunction**

Two separate semiconductors:

\[
\begin{align*}
\text{n-type} & \quad \text{intrinsic} \\
\text{Ionized Donors} & \quad \text{Electrons} \\
\text{Band Offset} & \quad \text{Energy} \; U = -eV \\
\end{align*}
\]

After forming a junction:

Ionized Donors

Electrons

Energy \( U = -eV \)

\( V = \text{Electrostatic Potential} \)
**Modulation Doping:** Achieve high electron density without scattering of electrons by donors. Record mobilities of $>10^6$ cm$^2$/Vs. Example: GaAlAs / GaAs

Used for creating a two-dimensional electron gas and for the quantum Hall effect.

### 2) Semiconductor – Metal: Schottky Barrier

Separate semiconductor and metal:

After forming a junction:

- **Schottky Barrier (n-type)**
- **Electrons trapped in interface states**
- **Depletion width $\sim 1/\sqrt{N_D^+}$**

### 3) Metal Oxide Semiconductor (MOS):

For a p-type semiconductor = nMOS (conducting via n-type carriers using **inversion**)

**Flatband**

**Accumulation**

**Inversion**

*Field Effect Transistor (MOSFET) see p. 7.*
Calculating Carrier Densities and Band Diagrams

A. Homogeneous Semiconductor, Static Equilibrium

Two variables: \( n_h \) = Hole Density, \( n_e \) = Electron Density

Two equations:

1. **Charge Neutrality:**
   \[
   n_h - n_e + N_D^+ - N_A^- = 0
   \]
   \( N_D^+ \) = Ionized Donor Density
   \( N_A^- \) = Ionized Acceptor Density

   Determines the position of the Fermi level in the gap.

2. **Mass Action:**
   \[
   n_h \cdot n_e = \text{const.} = [n_{\text{int}}(T)]^2
   \]
   \( n_{\text{int}} \) = Intrinsic Carrier Density

   Chemistry analog: \( H^+ + OH^- \Leftrightarrow H_2O \)
   \( h^+ + e^- \Leftrightarrow \text{photon} \)

   \( \Rightarrow \) Recombination rate \( \sim n_h \cdot n_e \) (\( e^- \) recombines with \( h^+ \))
   \( \Leftarrow \) Generation rate \( \sim \exp[-E_g/k_BT] \) (Thermal photon creates \( e^- h^+ \) pair)

In equilibrium the rates for the forward and back reaction are equal.
Driving up \( n_h \) reduces \( n_e \), because the extra holes take away electrons by recombination.

B. Inhomogeneous Semiconductor (Junctions), Static Equilibrium

Charge neutrality is gone. The space charge induced by the flow of carriers across a junction decays over a Debye screening length \( \sim 1/\sqrt{n} \). It is an analog to the Thomas-Fermi screening length in metals and the London penetration depth in superconductors.

One needs to solve two equations simultaneously (= self-consistently) by iteration:

1. **Poisson Equation:**
   \[
   \frac{d^2 V}{dz^2} = -\frac{\rho(z)}{\varepsilon \varepsilon_0}
   \]
   \( \rho = \text{Total Charge Density} \)
   \( V = \text{Electrostatic Potential} \)

   \[U(z) = q V(z)\]
   \( q = \text{Charge} \)
   \( = \text{Potential Energy} \)

   \( D^-(E) = \text{Density of States for Conduction Band + Acceptors} \)

   \( D^+(E) = \text{Density of States for Valence Band + Donors} \)

2. **Fermi Dirac Distribution:**
   \[
   V(z) \Rightarrow \rho(z)
   \]
   \[
   \rho(z) = + e \cdot n^-(z) - e \cdot n^+(z)
   \]
   \[
   n^-(z) = \int f_e(E) \cdot D^-[E+U(z)] \, dE
   \]
   \[
   n^+(z) = \int [1-f_e(E)] \cdot D^+[E+U(z)] \, dE
   \]
   \[
   f_e(E) = \frac{1}{\exp[(E-E_F)/k_BT] + 1}
   \]
C. Junction with Bias Voltage $V$, Dynamic Equilibrium

**Reverse Bias:**

- Minority carriers are generated thermally and drift down the barrier, independent of $V$.

**Forward Bias:**

- Majority carriers from dopants diffuse against the barrier ($E_g - eV$), depends exponentially on $V$.

Two opposing currents, matched at $V=0$

**Generation (Drift) Current:**

**Minority** carriers are generated thermally and drift down the barrier, independent of $V$.

**Recombination (Diffusion) Current:**

**Majority** carriers from dopants diffuse against the barrier ($E_g - eV$), depends exponentially on $V$.

$I = -1 + e^{eV/kT}$

**Rectifying Diode**

- Reverse Bias
- Zero Bias
- Forward Bias

**Photodiode**

- Reverse Bias
- Photon creates $e^-h^+$ pair

**Solar Cell**

- Forward Bias

**Light Emitting Diode (LED)**

- Forward Bias
- $e^-h^+$ pair recombines into a photon
Figure 10-38 Summary of the performance improvements in LEDs over the span of their existence. The ■ marks the current performance of small-molecule OLEDs; the ⚫ marks that of the polymer OLEDs. A few performance benchmarks are indicated on the vertical axis.
2) **Field Effect Transistor (FET):** Gate Voltage controls channel conductivity

nMOS:

![nMOS Diagram]

\[ \text{Off: } V_{\text{Gate}} \approx 0 \]

\[ \text{On: } V_{\text{Gate}} > 0 \]

This cut: Source – Channel – Drain = back-to-back diodes

Other cut: Gate – Oxide – Channel (p. 3)

**CMOS** (Complementary Metal Oxide Semiconductor)

Back-to-back nMOS and pMOS with common gate.

Low power consumption, draws current only during switching.
DRAM (Dynamic Random Access Memory)
Single field effect transistor + storage capacitor.
Needs to be refreshed due to leakage current.

CCD (Charge-Coupled-Device)
Array of MOS capacitors that store light-induced charge and pass it on along the array for readout.

Dark                      Illuminated

HEMT  (High Electron Mobility Transistor)  =  MODFET  (Modulation-Doped FET)
Modulation doping achieves high mobility in the channel by putting the dopants into an adjacent layer and let the electrons spill over. Fastest transistors (GaAlAs/GaAs, SiGe/Si).

Quantum Well Laser
A semiconductor with smaller band gap is inserted at the pn junction of a LED. Traps both electrons and holes and gives them time to recombine into photons. Concentrates the density of states in quantum levels.