Measuring the quantum numbers $E, k$ of electrons in a solid

The quantum numbers $E$ and $k$ can be measured by angle-resolved photoemission. This is an elaborate use of the photoelectric effect, which was explained as quantum phenomenon by Einstein in 1905.

Energy and momentum of the emitted photoelectron are measured.

Energy conservation:

\[ E_{\text{final}} = E_{\text{initial}} + h \nu \]

\[ h \nu = hf = E_{\text{photon}} \]

Momentum conservation:

\[ k_{\parallel,\text{final}} = k_{\parallel,\text{initial}} + G_{\parallel} \]

\[ k_{\text{photon}} \approx 0 \]

Only $k_{\parallel}$ is conserved (surface!)
Photoemission setup:

Photoemission process:

Photoemission spectrum:

\[ \Phi = h\nu - W \]
Measuring $E(k)$ of Cu by angle-resolved photoemission

$E_{\text{final}} \rightarrow E$

$\theta, \phi \rightarrow k_\parallel$

$h\nu \rightarrow k_\perp$

2D surface state $S_1$ is independent of $k_\perp, h\nu$. 
Photoemission
(PES, UPS, ARPES)

- Measures an “occupied state” by creating a **hole**
- Determines the complete set of quantum numbers
- Probes several atomic layers (surface + bulk)
Inverse Photoemission
(BIS, IPES, KRIPES)

- Measures an “empty state” by filling it with an electron
- Very low cross section (down from photoemission by $\alpha^2 \approx 5 \times 10^{-5}$)
Surface states on Si(111)7x7
Two-Dimensional Electron Gas (2DEG)

Angle-resolved photoemission measures $E, k_x, k_y$.

Two cuts:

- Fermi “surface”: $I(k_x, k_y)$
- Band dispersion: $I(E, k_x)$

Free electron: $E(k) = \frac{\hbar^2 k^2}{2m} = \text{Paraboloid}$
Surface electrons on Si(111)-Ag

Fermi “surface”

Doping by extra Ag atoms

e^-/atom: 0.0015 0.012 0.015 0.022 0.086

Band dispersion
Fermi “surfaces” of two- and one-dimensional electrons

$2D + \text{ superlattice} \rightarrow 1D$
E, k multidetection: Energy bands on a TV screen

Calculated $E(k)$ vs. Measured $E(k)$

$E_F$ vs. $E$ for Ni

$E_F$ at $k$-points: $\Gamma$, $K$, $X$

Electrons within $2k_B T$ of the Fermi level $E_F$ determine magnetism, superconductivity, ...

$E, \theta$ multidetection: 50x50 = 2500 channels