Superconductivity

In a superconductor, electrons form **pairs** which are **bound** by an **energy** $E_g$ (the **superconducting gap**). The pairs are held together by exchanging **phonons** (see the $k$-space diagram below). As analog, an electron bound to a positron ($e^+$) is shown, where the Coulomb attraction can be described by the exchanging photons. More generally, forces are created by exchanging bosons. Pairs in high temperature superconductors are held together by exotic bosons that have yet to be identified. In real space, phonons may be visualized by the movement of positive ions (= atoms minus the metallic valence electrons). These pile up inside the negative electron pair and mediate an attraction between the electrons. Visualizing the pairs is difficult in real space, but easy in $k$-space.

Real space ($r$)      Reciprocal space ($k$)

Cooper pair:

Analog: $e^-e^+$:

The picture is complicated by the fact that there are $10^7$ other **pairs** within the **pair diameter** $\xi$ (= **coherence length**). All of them attract ions that contribute to the electron-electron attraction. And the large size of the pairs reduces their Coulomb repulsion. The value of $\xi$ can be estimated from the Fermi velocity $v_F \approx 10^6$ m/s and the phonon vibration period $T_{ph} \approx 10^{-12}$ s: $\xi \approx v_F \cdot T_{phonon} \approx 1 \mu m$

Essentially, the positive ions take a long time to get going, and by that time the electron has already moved away by a distance $\xi$.

**Evidence for pairs**: The magnetic flux $\Phi$ is quantized in units of $2e$, not $e$: $\Phi_0 = \hbar/2e$

**Evidence for phonons**: The critical temperature $T_c$ is **proportional to** $M^{-\frac{1}{2}}$ for different **isotopes** of an element ($M =$ atomic mass), like the phonon frequency $\omega_{ph}$ (Lect. 11).
Quantum numbers of electron pairs (in addition to the pair energy $-E_g$)

<table>
<thead>
<tr>
<th>Quantum number</th>
<th>$e_1$</th>
<th>$e_2$</th>
<th>pair</th>
</tr>
</thead>
<tbody>
<tr>
<td>Momentum: $p=\hbar k$</td>
<td>$+k$</td>
<td>$-k$</td>
<td>0</td>
</tr>
<tr>
<td>Spin: $s$</td>
<td>$\frac{1}{2}$</td>
<td>$\frac{1}{2}$</td>
<td>0</td>
</tr>
<tr>
<td>$m_s$</td>
<td>$+\frac{1}{2}$</td>
<td>$-\frac{1}{2}$</td>
<td>0</td>
</tr>
</tbody>
</table>

"singlet" pairing ($s=0$)

"s-wave" pairing ($l=2$ "d-wave", HiTc)

Why is the resistance zero in a superconductor?

Electrical resistance is caused by electrons losing energy by scattering. The electron pairs are all in their ground state and cannot lose energy. When accelerated to carry a current the pairs acquire kinetic energy. However, they need to acquire at least a kinetic energy equal to the energy gap $E_g$ to lose energy by breaking a pair (compare the increase of the mean free path of low-energy electrons in solids in Lect. 19, Slide 12).

Is the resistance really zero?

Experimental answer: A lower limit of $10^5$ years has been established for the decay time $\tau$ of the current $I$ in a superconducting loop ($10^{-5}$ precision over a year).

Use $I = \exp(-t/\tau)$, $\tau = L/R$, $L =$ inductance, $\Rightarrow$ resistance $R$.

Theoretical answer: The decay time $\tau$ is given by the inverse of the transition rate, which is determined by the product of an attempt frequency $f$ and a probability $p$ per attempt (compare superparamagnetism. Lect. 23, Slide 9): $\tau = (f \cdot p)^{-1}$

Since superconductivity is mediated by phonons, a typical phonon frequency $f \approx 10^{12}$ Hz is appropriate for the attempt frequency. The probability is given by the Boltzmann factor for breaking all $10^7$ pairs within a coherence length: $p \approx \exp(-10^7E_g/k_B T)$. If a single pair remained, it would carry the current with zero resistance and pass it on to another pair in an adjacent coherence volume. $E_g$ is comparable to $k_B T_c$ ($T_c =$ superconducting transition temperature). If the temperature is low enough ($T \ll T_c$), the decay time becomes $\tau \approx 10^{-12} s \cdot \exp(10^7) \approx 10^{4342915}$ times the age of the universe.
What kills superconductivity?

Need to provide enough energy to break up all pairs within a coherence length:

1. Thermal energy: \( \rightarrow \) Critical temperature \( T_c \)
2. Kinetic energy: \( \rightarrow \) Critical current density \( J_c \)
3. Magnetic energy: \( \rightarrow \) Critical field \( H_c \)

What drives superconductivity?

The density of states at the Fermi level \( D(E_F) \) plays a critical role. These are the most unstable electrons in a solid, and they easily rearrange themselves to lower their energy. Electrons near \( E_F \) are able to lower their energy when the superconducting gap opens up (lightly hatched area below). Each electron gains the energy \( \Delta \), and a pair gains \( 2\Delta = E_g \).

The number of electrons involved is given by the hatched area \( \frac{1}{2} E_g \cdot D(E_F) \).

\[
\begin{align*}
T > T_c: & \quad \text{Normal metal} \\
T < T_c: & \quad \text{Superconductor}
\end{align*}
\]

This mechanism for energy reduction resembles the Stoner model for ferromagnetism, where the exchange splitting \( \delta E_{\text{ex}} \) plays the role of the gap \( E_g \) (Lect. 21, p. 3). \( D(E_F) \) plays again a key role, like in the Stoner criterion. This competition for unstable electrons at \( E_F \) explains why superconductivity and magnetism do not get along (compare Lect. 21, p. 4): Superconductivity is destroyed by a magnetic field (the critical field \( H_c \)), and a magnetic field is expelled from a superconductor by the Meissner effect (Lect. 29, Slides 2,3).
Measuring the gap $E_g$

1. **Infrared Absorption**: The absorption coefficient is measured versus the energy of infrared or microwave photons, analogous to a gap measurement in a semiconductor. (Only the energy scale is meV instead of eV). In the superconducting state, photons cannot be absorbed when their energy is less than $E_g$. (Lect. 29, Slide 4)

2. **Tunneling**: Electrons tunnel from a metal through an insulator into a superconductor. Current-versus-voltage $I(V)$ curves are measured. The current vanishes for $|V|<\Delta/e$. The derivative $dI/dV$ is related to the density of states $D(E)$. (Lect. 29, Slide 5)

3. **Photoemission**: The number of emitted photoelectrons versus their final state energy $E_{\text{fin}}$ replicates the density of states $D(E)$. A superconductor does not have any states in the energy region $E_F-\Delta < E < E_F$. Photoemission can measure the $k$-dependence of $E_g$ and thereby determine the orbital angular momentum $l$ of the pairs. That is particularly important for high temperature superconductors, which are d-wave (Lect. 29, Slide 6). In all these experiments, the normal state just above $T_c$ is compared to the superconducting state below $T_c$ to eliminate all effects other than superconductivity.

High Temperature Superconductors (HiTc)
Superconductivity occurs in the $\text{Cu}^{2+}-\text{O}^{2-}$ planes embedded into an ionic lattice.

The superconducting carriers are holes introduced by doping.

The big open question is the nature of the boson that gives rise to pairing according to the diagram on p. 1 (magnon?, phonon?, complex mixture of those?).