Effect of the spin-dependent electromagnetic forces on low-energy proton-proton scattering

L. D. Knutson* and D. Chiang
University of Washington, Seattle, Washington 98195
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The effects of electromagnetically induced spin-dependent potentials on the analyzing power for low-energy proton-proton scattering are calculated using the Coulomb-distorted-wave Born approximation. The amplitudes which arise from these potentials are nearly in phase with the usual Coulomb amplitude and as a result the potentials have virtually no effect on the analyzing power.

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In the scattering of particles with spin, polarization effects can arise from purely electromagnetic interactions. For example, the interaction of the magnetic moment of a projectile with the Coulomb field of the target produces a spin-dependent force which can result in a nonzero polarization or analyzing power. In proton-proton scattering at energies of a few hundred MeV these effects are significant. Thus if one wishes to isolate the nuclear effects, it is necessary to correct for the electromagnetic processes. At lower energies ($E_p < 50$ MeV) corrections have been unnecessary since the errors in the measured analyzing powers were much larger than the predicted electromagnetic effects. However, recent polarized beam experiments[1,2] have produced highly accurate analyzing power data at 10 and 16 MeV. These new measurements make it necessary to reconsider the importance of the spin-dependent electromagnetic potentials. In this paper we calculate the effects of these potentials using approximations which are valid at low energies.

From relativistic theories of the proton-proton interaction (see Ref. 3) one finds that, to first order in the fine-structure constant, the proton-proton potential contains two spin-dependent terms of electromagnetic origin: a spin-orbit potential,

$$V_{ls} = -8\mu_0(\mu_T - \tfrac{1}{4}\mu_0)r^{-3}\vec{l}\cdot\vec{S}\,, \tag{1}$$

and a tensor potential,

$$V_T = -\mu_T{}^2 r^{-3} S_{12}\,, \tag{2}$$

where

$$\mu_0 = e\hbar/2mc\,, \tag{3}$$

$$\mu_T = 2.792\,76\,\mu_0\,, \tag{4}$$

$$\vec{S} = \tfrac{1}{2}(\vec{\sigma}_1 + \vec{\sigma}_2)\,, \tag{5}$$

$$S_{12} = 3(\vec{\sigma}_1\cdot\hat{r})(\vec{\sigma}_2\cdot\hat{r}) - \vec{\sigma}_1\cdot\vec{\sigma}_2\,, \tag{6}$$

and where m is the proton mass. The spin-orbit potential describes the interaction of the magnetic moment of each proton with the Coulomb field of the other and includes a correction for the Thomas precession, while the tensor potential corresponds to the moment-moment interaction.

In order to calculate the effect of these spin-dependent potentials we make use of the distorted-wave Born approximation for the scattering from two potentials. For a potential of the form

$$U = V + V' \tag{7}$$

the elastic scattering wave function, $\psi^{(+)}$, satisfies an integral equation

$$\psi^{(+)}(\vec{k},\vec{r}) = \phi^{(+)}(\vec{k},\vec{r}) - \int G^{(+)}(\vec{r},\vec{r}\,')V'(\vec{r}\,')\psi^{(+)}(\vec{k},\vec{r}\,')d^3r'\,. \tag{8}$$

Here $\phi^{(+)}$ is a wave function for scattering from V alone, and G is a Green's function corresponding to V. The distorted-wave Born approximation is obtained by replacing ψ on the right-hand side of Eq. (8) by ϕ.

We take V to be the sum of the Coulomb and nuclear potentials and treat the potentials of Eqs. (1) and (2) as perturbations. Now for low energies the nuclear potential has a large effect only in the $l=0$ partial wave. Since the Pauli principle restricts the $l=0$ scattering to the singlet spin state (where the spin-dependent forces vanish), it is reasonable to neglect the effect of the nuclear forces when evaluating the integral in Eq. (8). We therefore replace $\psi^{(+)}$ in the integral by a pure Coulomb wave function, $\phi_C^{(+)}$, and take $G^{(+)}$ to be the usual Coulomb Green's function. For large r, $G^{(+)}$ becomes

$$G^{(+)}(\vec{r},\vec{r}\,') \xrightarrow[r\to\infty]{} \frac{m}{4\pi\hbar^2}\frac{1}{r}\exp\{i[k'r - \eta\ln(2k'r)]\} \times \phi_C^{(-)*}(\vec{k}\,',\vec{r}\,')\,, \tag{9}$$

where η is the Coulomb parameter

$$\eta = \tfrac{1}{2}me^2/\hbar^2 k\,. \tag{10}$$

In order to describe the scattering of identical particles, we must ensure that $\psi^{(+)}$ is properly antisymmetrized. This is done by simply choosing $\phi_C^{(+)}$ to be an antisymmetric wave function with the correct boundary conditions. On the other hand, the Green's function [i.e., the wave function $\phi_C^{(-)}$ in Eq. (9)] should not be antisymmetrized. Thus we find that the scattering amplitude is given by

$$f(\vec{k},\vec{k}')=f_C(\vec{k},\vec{k}')+f_N(\vec{k},\vec{k}') - \frac{m}{4\pi\hbar^2}\int \phi_C^{(-)*}(\vec{k}',\vec{r}')V'(\vec{r}')\phi_C^{(+)}(\vec{k},\vec{r}')d^3r' , \tag{11}$$

where f_C and f_N are the Coulomb and nuclear scattering amplitudes, respectively, and where it is understood that $\phi_C^{(+)}$ is to be antisymmetrized but that $\phi_C^{(-)}$ is not.

Since the matrix elements of $\vec{l}\cdot\vec{s}$ and S_{12} between singlet spin states are zero we consider only the triplet states. In this case the appropriate Coulomb amplitude is

$$f_C(k,k')=-\frac{\eta}{k}\left\{\frac{1}{1-\mu}\exp[-i\eta\ln\tfrac{1}{2}(1-\mu)] - \frac{1}{1+\mu}\exp[-i\eta\ln\tfrac{1}{2}(1+\mu)]\right\}e^{2i\alpha_0} , \tag{12}$$

where

$$\mu=\hat{k}\cdot\hat{k}' . \tag{13}$$

For the Coulomb wave functions we write

$$\phi_C^{(+)}(\vec{k},\vec{r})=(8\pi/kr)\sum_{\text{odd } l}\sum_m i^l e^{i\alpha_l}F_l(kr)Y_l^m(\hat{r})Y_l^{m*}(\hat{k}) , \tag{14}$$

$$\phi_C^{(-)*}(\vec{k},\vec{r})=(4\pi/kr)\sum_{\text{all } l}\sum_m i^{-l}e^{i\alpha_l}F_l(kr)Y_l^{m*}(\hat{r})Y_l^m(\hat{k}) . \tag{15}$$

Here α_l is the usual Coulomb phase shift, and F_l is the regular solution to the Coulomb radial wave equation.

We now consider the spin-orbit potential in Eq. (1). We must evaluate the matrix element

$$\langle\sigma'|f_{ls}|\sigma\rangle=C_{ls}\int\phi_C^{(-)*}(\vec{k}',\vec{r})r^{-3} \times\langle\chi_1^{\sigma'}|\vec{l}\cdot\vec{s}|\chi_1^{\sigma}\rangle\phi_C^{(+)}(\vec{k},\vec{r})d^3r , \tag{16}$$

where $|\chi_1^\sigma\rangle$ is a triplet spin function and where

$$C_{ls}=2m\mu_0(\mu_T-\tfrac{1}{4}\mu_0)/\pi\hbar^2 . \tag{17}$$

We choose a coordinate system which has its z axis along $\vec{k}$ and its y axis along $\vec{k}\times\vec{k}'$. It is then straightforward to show that

$$\langle\sigma'|f_{ls}|\sigma\rangle=8\pi C_{ls}\langle\chi_1^{\sigma'}|iS_y|\chi_1^\sigma\rangle \times\sum e^{i\alpha_l}(2l+1)M_{ll}^{-3}P_l^1(\mu) \tag{18}$$

where

$$M_{ll'}^{-3}=k^{-2}\int_0^\infty r^{-3}F_l(kr)F_{l'}(kr)dr \tag{19}$$

and

$$P_l^\lambda=(1-\mu^2)^{\lambda/2}\frac{d^\lambda}{d\mu^\lambda}P_l(\mu) . \tag{20}$$

From Ref. 4 we find

$$M_{ll}^{-3}=[2l(l+1)(2l+1)]^{-1} \times\left(2l+1-\eta\pi+1+\eta\pi\coth\eta\pi-2\eta^2\sum_{p=0}^{l}\frac{1}{p^2+\eta^2}\right) . \tag{21}$$

In principle, one could evaluate f_{ls} by summing Eq. (18) term by term with a computer. However, the sum converges too slowly for this to be a practical method. Instead we split f_{ls} into two pieces,

$$\langle\sigma'|f_{ls}|\sigma\rangle=[f_1(\mu)+f_2(\mu)] \times\langle\chi_1^{\sigma'}|iS_y|\chi_1^\sigma\rangle , \tag{22}$$

where f_1 is defined as

$$f_1(\mu)=4\pi C_{ls}\sum_{\text{odd } l}e^{2i\alpha_l}(2l+1)[l(l+1)]^{-1}P_l^1(\mu) . \tag{23}$$

This separation is convenient because f_2 converges rapidly enough to be summed term by term, while f_1 can be summed analytically:

$$f_1=4\pi C_{ls}e^{2i\alpha_0}(1-\mu^2)^{-1/2} \times\{\exp[-i\eta\ln\tfrac{1}{2}(1-\mu)]+\exp[-i\eta\ln\tfrac{1}{2}(1+\mu)]-1\} . \tag{24}$$

If one were to use the plane-wave Born approximation to calculate the scattering amplitude these results would be somewhat modified. The plane-wave result is obtained by setting $f_2=0$ and setting $\alpha_0=\eta=0$ in Eq. (24). This has little effect on the magnitude of f_{ls}, but does change the phase (see below). Ebel and Hull[5] have used the plane-wave Born approximation to calculate the phase shifts resulting from the spin-orbit potential, but have included the effect of the Coulomb potential by including factors $\exp(2i\alpha_l)$ in the partial wave expansion for f_{ls}. Using this approach they obtain precisely the result in Eq. (24); however, the term f_2 is missing. At energies near 10 MeV, this is not serious since $|f_2|$ is about an order of magnitude smaller than $|f_1|$.

We now consider the tensor potential defined in Eq. (2). Using an approach similar to that de-

$$\langle\sigma'|f_T|\sigma\rangle=(4\sqrt{10}m\mu_T{}^2/\hbar^2)\sum_{\text{odd } l}\sum_{l',\lambda}(2l+1)i^{l-l'}[(l'-\lambda)!/(l'+\lambda)!]^{1/2}e^{i(\alpha_l+\alpha_{l'})}M_{ll'}{}^{-3}\langle l0,2\lambda|l'\lambda\rangle\langle l0,20|l'0\rangle P_{l'}^{\lambda}(\mu)\,, \tag{25}$$

where $M_{ll}{}^{-3}$ is given in Eq. (19) and[4]

$$M_{l,l+2}{}^{-3}=M_{l+2,l}{}^{-3}=(6|l+1+i\eta||l+2+i\eta|)^{-1}. \tag{26}$$

$$A=\text{Im}(f_1^*f_C)/(\tfrac{3}{4}|f_C|^2+\tfrac{1}{4}|f_{ss}|^2). \tag{27}$$

Here f_{ss} is the scattering amplitude for the singlet state. At $\theta_{\text{c.m.}}=30°$, for example, $|f_1|\simeq 0.02\,|f_C|$ and $|f_{ss}|\simeq 4|f_1|$. Thus one might expect A to

*Present address: Department of Physics, University of Wisconsin, Madison,Wisconsin 53706.

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