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Delta-shell method for approximate finite-range distorted-wave Born-approximation calculations

L. D. Knutson
University of Wisconsin, Madison, Wisconsin 53706
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The application of the delta-shell method to finite-range distorted-wave Born-approximation calculations for low-energy (d,p) reactions is discussed. It is shown that this method is superior to the commonly used local-energy approximation as far as calculational accuracy is concerned.

[NUCLEAR REACTIONS DWBA theory, finite-range effects.]

It has long been realized that the zero-range distorted-wave Born-approximation (DWBA) theory[1] is adequate for predicting the general shape and magnitude of the differential cross section for (d,p) reactions at low energies. However, for some of the polarization observables (in particular for the tensor analyzing powers) the theory is not successful. It is now known[2-4] that the deuteron D state, which is completely neglected in a zero-range calculation, has a very large effect on the tensor analyzing powers. For this reason, and for other reasons as well, it is often desirable to include finite-range effects in a DWBA calculation.

Exact finite-range DWBA calculations still present a formidable computational problem, since a large amount of computing time is required when the finite-range integral is evaluated in a straightforward manner. Thus it is often advantageous to make use of some approximation technique. One such technique, which has been used extensively in the past, is an extension of the local-energy ap-

where we have adopted the notation of Ref. 7. In this expression χ_p and χ_d are optical model wave functions for the proton and deuteron channels respectively, ψ_n is the wave function of the bound neutron, v_{np} is the neutron-proton potential, and J is a constant. The quantity ϕ_d in Eq. (1) is the internal wave function of the deuteron which may be written in the form

$$\phi_d^m(\vec{r}) = \frac{1}{r}\left[u(r)\mathcal{Y}^m_{101} - w(r)\mathcal{Y}^m_{121}\right] . \qquad (2)$$

Here $u(r)$ and $w(r)$ are the usual S- and D-state radial wave functions and $\mathcal{Y}^m_{jls}$ is a spin-angle function as defined in Ref. 7. By making use of the Schrödinger equation for ϕ_d it is straightforward to show that the product $v_{np}\phi_d$ may be written in the form

$$v_{np}(\vec{r})\phi_d^m(\vec{r}) = v_0(r)\mathcal{Y}^m_{101} - v_2(r)\mathcal{Y}^m_{121} , \qquad (3)$$

where

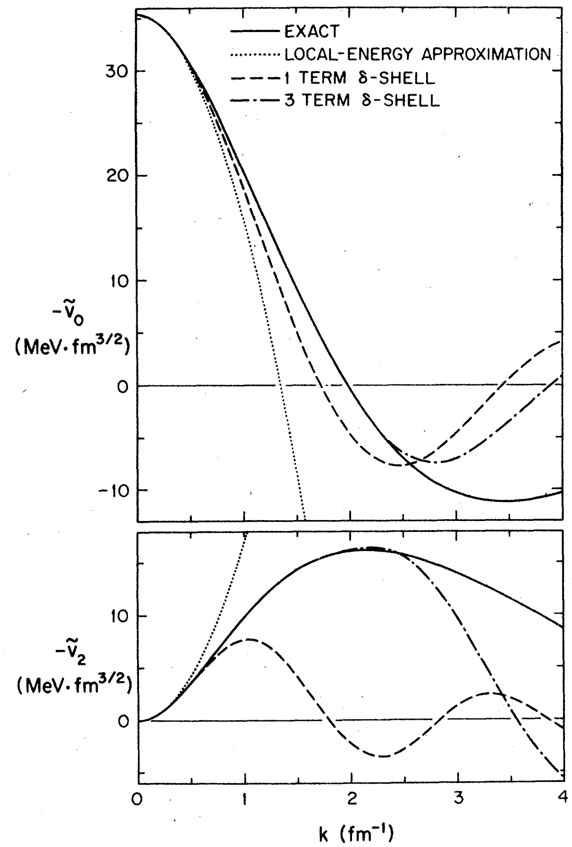

FIG. 1. S- and D-state parts of $v_{np}\phi_d$ in momentum space. The solid curves show the functions calculated from the soft-core nucleon-nucleon potential of Reid (Ref. 8). The remaining curves represent various approximations.

ψ_n in Eq. (1). Equivalently, one may argue that because of their short range the functions v_l contain high-momentum components, whereas χ_p, χ_d, and ψ_n consist primarily of low-momentum components. Therefore we may look for approximations in which v_l is replaced by some convenient function. Such an approximation will be reasonable provided that the low-momentum behavior of v_l does not change appreciably.

In momentum space the S- and D-state parts of $v_{np}\phi_d$ are given by the functions

$$\tilde{v}_l(k) = \int_0^\infty v_l(r) j_l(kr) r^2 dr \, , \tag{5}$$

where j_l is a spherical Bessel function. The momentum space functions calculated from the Reid soft-core potential[8] are shown by the solid curves in Fig. 1.

We may now compare the various approximation techniques by noting how accurately the functions $\tilde{v}_l(k)$ are reproduced for small values of k. First we consider the zero-range approximation in which

TABLE I. δ-shell parameter values.

Number of terms	a_0 (fm)	b_0 (MeV fm$^{-\frac{1}{2}}$)	a_2 (fm)	b_2 (MeV fm$^{-\frac{1}{2}}$)
1	1.828	-10.568	3.209	-2.419
3	1.609	-12.91	1.611	-21.56
	3.875	-0.1176	3.717	-0.4783
	7.284	-1.697 X 10^{-3}	7.088	-7.889 X 10^{-3}

$v_{np}\phi_d$ is replaced by a delta function, i.e.

$$v_{np}(\vec{r})\phi_d(\vec{r}) \simeq D_0 \delta(\vec{r}) \, , \tag{6}$$

where

$$D_0 = \int v_{np}(\vec{r})\phi_d(\vec{r}) d^3r \, . \tag{7}$$

In momentum space, this amounts to setting

$$\begin{aligned} \tilde{v}_0(k) &\simeq \tilde{v}_0(0) = D_0/\sqrt{4\pi} \, , \\ \tilde{v}_2(k) &\simeq \tilde{v}_2(0) = 0 \, . \end{aligned} \tag{8}$$

As one can see from Fig. 1, this approximation is quite inaccurate for $k \gtrsim 0.5$ fm^{-1}.

In the local-energy approximation,[5] one improves on the zero-range approximation by expanding $\tilde{v}_l(k)$ in a power series and keeping terms up to order k^2. Adopting the conventional notation[5,7]

$$\begin{aligned} \tilde{v}_0(k) &\simeq \tilde{v}_0(0)(1 - k^2/\beta^2) \, , \\ \tilde{v}_2(k) &\simeq \tilde{v}_0(0) D_2 k^2 \, . \end{aligned} \tag{9}$$

This approximation is represented by the dotted curves in Fig. 1, and the corresponding parameter values are

$$\begin{aligned} \tilde{v}_0(0) &= -35.30 \text{ MeV fm}^{3/2} \, , \\ \beta &= 1.340 \text{ fm}^{-1} \, , \\ D_2 &= 0.484 \text{ fm}^2 \, . \end{aligned} \tag{10}$$

We now turn to the δ-shell approximation.[6] Here one sets

$$v_l(r) \simeq b_l \delta(r - a_l) \, , \tag{11}$$

which corresponds to

$$\tilde{v}_l(k) \simeq b_l a_l^2 j_l(ka_l) \, . \tag{12}$$

As yet, the constants a_l and b_l are unspecified, but from the preceding discussion it is clear that these parameters should be chosen to reproduce as closely as possible the low-momentum behavior of $\tilde{v}_l(k)$. For example, we may choose a_l and b_l so that the first two terms in the power series expansions of $\tilde{v}_l(k)$ and $b_l a_l^2 j_l(ka_l)$ are identical. The parameter values corresponding to this choice are given in Table I, and the resulting momentum space functions are shown by the dashed curves in

Fig. 1.

As one can clearly see in Fig. 1, the δ-shell approximation is superior to the local-energy approximation as far as calculational accuracy is concerned. This is particularly true for the D-state function. For example, at $k=0.5$ fm^{-1} (which corresponds to an n-p relative energy of about 10 MeV) the δ-shell approximation is within 2% of the correct value (for the D-state function) whereas the local-energy approximation is off by about 18%.

It is quite clear that one can improve on the simple δ-shell approximation by making use of an expansion which contains more than one δ-shell term:

$$v_l(r) \simeq \sum_{i=1}^{N} b_{l,i}\,\delta(r-a_{l,i}). \qquad (13)$$

The use of an N term expansion in a DWBA calculation would increase the computation time by about a factor of N, but would not otherwise affect the complexity of the calculation. To illustrate the improved accuracy which can be obtained, the dot-dashed curves in Fig. 1 show momentum space functions corresponding to $N=3$. The parameter values are given in Table I. For the D-state function, the three-term result is within 1% of the correct value out to $k=2$ fm^{-1}, which corresponds to an n-p relative energy of over 160 MeV. Thus the three-term expansion should be quite sufficient for most calculations at low energies.

tributions to the integral in Eq. (1) can be neglected whenever $r>r_{\max}$. In practice one might choose $r_{\max}=5$ fm. On the surface, this choice seems reasonable, since at 5 fm the radial functions $v_l(r)$ have fallen to less than 0.1% of their peak values. However, the use of such a cutoff can produce substantial errors for calculations which include the D state, because the low-momentum behavior of $\tilde{v}_2(k)$ is extremely sensitive to the long-range part of $v_2(r)$. For example, with $k=0.5$ fm^{-1}, one finds that

$$\int_0^{5\text{ fm}} v_2(r)\,j_2(kr)\,r^2dr \bigg/ \int_0^{\infty} v_2(r)\,j_2(kr)\,r^2dr = 0.95\ . \qquad (14)$$

(This ratio is roughly independent of k for small k.) Thus the D-state effects predicted from finite-range calculations using a cutoff of 5 fm are likely to be in error by as much as 5%. To obtain 1% accuracy one would need to extend the integral out to $r_{\max}=8$ fm.

To summarize, it has been shown that the low-momentum behavior of the function $v_{np}\phi_d$ is more accurately reproduced by the delta-shell approximation than by the local-energy approximation. In addition, we have seen that the momentum dependence of $v_{np}\phi_d$ can be accurately reproduced out to $k=2$ fm^{-1} by using a three-term delta-shell expansion. Finally, it was pointed out that the use of a cutoff at $r\simeq 5$ fm in a finite-range DWBA