$E_{c.m.} \sim 11.3$ MeV).

The above is speculation and it must be born in mind that in cases where strong absorption is present, like $^{12}C + ^{12}C$, predictions have been made of resonant behavior on a strictly statistical basis.[9] However, it is indicated that a simple interpretation with appealing consequences may be possible and that the understanding of the 19.3-MeV resonance may be learned from further experimentation.

We thank Professor H. Feshbach, A. K. Kerman, A. Bohr, and B. Mottelson for their interesting discussions of these data. We also acknowledge the help of Dr. J. Garrett and Mr. Tun Chin in running the Coriolis coupling and Hauser-Feshbach calculations for us.

*Work supported in part through funds provided by the Atomic Energy Commission under Contract No. AT(11-1)-3069.

†Alfred P. Sloan Foundation Fellow.

[1]E. Almqvist, D. A. Bromley, and J. A. Kuehner, Phys. Rev. Lett. 4, 515 (1960).

[2]J. R. Patterson, B. N. Nagorcka, G. D. Symons, and W. M. Zuk, Nucl. Phys. A165, 545 (1971).

[3]R. G. Stokstad, in *Europhysics Study Conference on Intermediate Processes in Nuclear Reactions, Plitvice Lakes, Yugoslavia, 1972, Proceedings*, edited by N. Cindro *et al.* (Springer, Berlin, 1973).

[4]R. E. Malmin, R. H. Siemssen, D. A. Sink, and P. P. Singh, Phys. Rev. Lett. 28, 1590 (1972); R. Stokstad, D. Shapira, L. Chua, P. Parker, M. W. Sachs, R. Wieland, and D. A. Bromley, Phys. Rev. Lett. 28, 1523 (1972); E. R. Cosman, A. Sperduto, T. M. Cormier, T. N. Chin, H. E. Wegner, M. J. LeVine, and D. Schalm, Phys. Rev. Lett. 29, 1341 (1972).

[5]L. R. Greenwood and R. E. Segel, in *Proceedings of the International Conference on Nuclear Physics, Munich, Germany, 1973*, edited by J. de Boer and H. J. Mang (North-Holland, Amsterdam, 1973), p. 543; D. Shapira, R. Stokstad, L. Chua, A. Gobbi, M. W. Sachs, R. Wieland, and D. A. Bromley, *ibid.*, p. 546.

[6]E. R. Cosman, K. Van Bibber, A. Sperduto, T. Cormier, T. N. Chin, and O. Hansen, Bull. Amer. Phys. Soc. 18, 565 (1973).

[7]P. Sperr, W. Assman, D. Evers, J. Hertel, K. Rudolph, and E. Spindler, in *Proceedings of the International Conference on Nuclear Physics, Munich, Germany, 1973*, edited by J. de Boer and H. J. Mang (North-Holland, Amsterdam, 1973), and to be published.

[8]Performed with the code SERENA by T. N Chin and T. M. Cormier, to be published.

[9]P. A. Moldauer, Phys. Rev. Lett. 18, 249 (1967).

Classical Description of the Deuteron *D*-State Effects in Sub-Coulomb (d, p) Reactions

L. D. Knutson, E. J. Stephenson, and W. Haeberli
*University of Wisconsin, Madison, Wisconsin 53706**
(Received 14 January 1974)

The effects of the deuteron D state on sub-Coulomb (d,p) reactions are explained in terms of a simple physical model. The model, which is based on a classical description of the reaction, makes it possible to understand the observed D-state effects for ^{208}Pb$(d,p)^{209}$Pb at 9 MeV.

In an earlier Letter[1] it was demonstrated that the deuteron D state has an important effect on some of the measurable quantities for (d, p) reactions. The effects are especially large for the tensor analyzing powers (T_{20}, T_{21}, T_{22}), which are a measure of the change in cross section which results when the incident deuteron beam is aligned.[2]

In this Letter we will present a model of the (d, p) reaction which is based on the concepts of classical physics. The purpose of this model is to provide a basis for understanding, in simple terms, why the deuteron D state affects the tensor analyzing powers for a (d, p) reaction.

The model which we propose is applicable to sub-Coulomb reactions. For sub-Coulomb energies, the reactions occur primarily outside the nucleus where the nuclear forces between the targets and projectiles have little influence on the reaction.[3] The special properties of sub-Coulomb reactions which make it possible to use a classical picture have been discussed in the literature.[3,4] Classical models have previously been used to explain the cross section[4,5] and the vector analyzing power[4] of sub-Coulomb (d, p) reactions. Models of this type are valuable because they provide physical insight which is often obscured in more formal treatments involv-

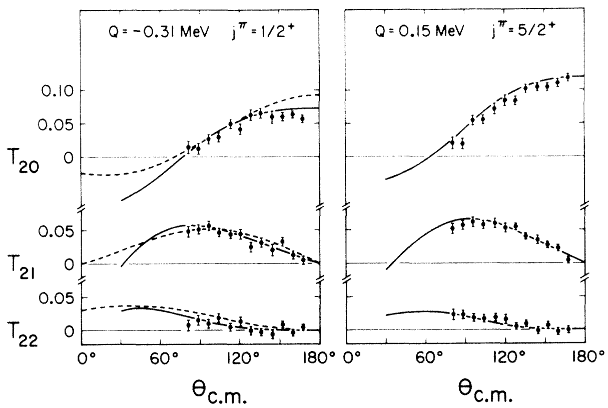

FIG. 1. Angular distributions of the three tensor analyzing powers for the reaction ^{208}Pb(d,p)^{209}Pb at 9 MeV. The solid curves are the result of a DWBA calculation (including the deuteron D state) in which the nuclear optical model potentials were set equal to zero. The dashed curves show the result of a calculation based on the classical model.

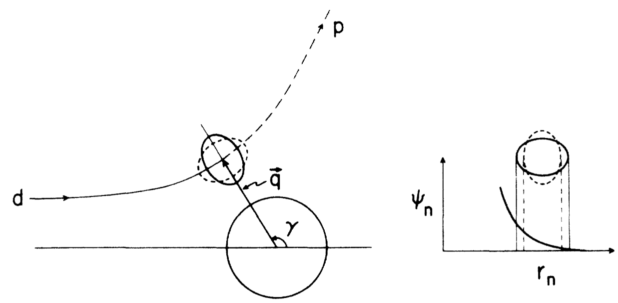

FIG. 2. Schematic representation of a sub-Coulomb (d,p) reaction. The incident deuterons and the outgoing protons follow classical Coulomb trajectories, and the neutron transfer occurs at the point $\vec{q}$. The ellipses represent the shape of the deuteron wave function. The right-hand side of the figure illustrates that reactions are most likely to occur when the deuteron spin is parallel or antiparallel to $\vec{q}$.

ing detailed computer calculations.

The validity of the model will be judged by comparing the predictions of the model with measured tensor analyzing powers and with calculations based on the distorted-wave Born approximation (DWBA). Measured angular distributions of the three tensor analyzing powers and the corresponding DWBA calculations (given by the solid curves) for two sub-Coulomb (d, p) transitions are shown in Fig. 1. The DWBA calculations, which include the deuteron D state,[6] were carried out assuming that the deuteron and proton are subject to Coulomb distortions only. Without the deuteron D state, the calculated tensor analyzing powers would be zero. Note that the angular dependence of the tensor analyzing powers is very simple and that the results for the two transitions are strikingly similar even though the final states have different spins. This leaves one with the impression that the reaction process cannot be very complicated.

The important features of the model which we propose are illustrated in Fig. 2. It is assumed that the incident deuteron and the outgoing proton follow classical Coulomb trajectories.[7] If the Q value of the reaction is zero (which is approximately true for the transitions shown in Fig. 1) the two particles will follow a single hyperbolic orbit, as shown in Fig. 2. The neutron capture is assumed to occur at the classical turning point of this orbit,[7] and, as a result, for a given deuteron energy and reaction angle, the neutron capture occurs at a unique point in space, denoted by the vector $\vec{q}$. For $Q=0$, the reaction angle θ is related to γ, the angle between $\vec{q}$ and the incident deuteron beam, by

$$\gamma = \tfrac{1}{2}(\pi + \theta). \tag{1}$$

We now consider the effects of the deuteron D state. The deuteron quadrupole moment is a direct consequence of the existence of the D state. In simple terms, the positive quadrupole moment means that the deuteron is stretched out along the direction of its spin. The solid- and dashed-line ellipses in Fig. 2 represent the "shape" of the deuteron wave function[8] for two orientations of the spin axis.

The basic assumption of our model is that the reaction is more likely to occur for $\vec{s}$ parallel or antiparallel to $\vec{q}$ (the solid-line ellipse) than for $\vec{s}$ perpendicular to $\vec{q}$ (the dashed-line ellipse). The justification for this assumption is illustrated on the right-hand side of Fig. 2. Outside the nucleus the bound-neutron wave function ψ_n falls off exponentially. When $\vec{s}$ lies along $\vec{q}$, the overlap between the deuteron wave function and the neutron wave function (and hence the reaction cross section) will be enhanced, because, for this orientation, the deuteron extends farther into the region where ψ_n is large. It should be emphasized that this enhancement results solely from the curvature of the neutron wave function.

From this simple picture of the reaction one can easily understand the effects of the deuteron D state. For example, in order to measure the vector analyzing power, one measures the change in cross section between spin up ($\vec{s}$ parallel to

$\vec{k}_{in}\times\vec{k}_{out}$) and spin down ($\vec{s}$ antiparallel to $\vec{k}_{in}\times\vec{k}_{out}$). Since the ellipses representing the shape of the deuteron are the same for these two spin directions, the D state has no effect on the vector analyzing power.

In order to understand how the D state affects the tensor analyzing powers, it is helpful to discuss how these quantities are measured. One begins with a deuteron beam which is "aligned." Although the term aligned is more general, here we shall use it to mean a beam for which all of the deuteron spins are either parallel or antiparallel to some axis, referred to as the spin alignment axis. For this type of beam, all of the deuterons are elongated along the direction of the alignment axis, in contrast to an unpolarized beam for which the deuterons are randomly oriented. With an aligned beam one can determine the tensor analyzing powers simply by measuring the relative cross section for different orientations of the alignment axis. For example, the analyzing power T_{20} can be measured by using a beam with the alignment axis parallel to the incident beam direction. For this particular orientation, the cross section measured with the aligned beam, σ_a, is related to the unpolarized cross section σ_u by[9]

$$\sigma_a=\sigma_u[1+2^{-1/2}T_{20}]. \qquad (2)$$

We shall now use the classical model to predict the behavior of T_{20}. According to the model, reactions which occur at forward angles correspond to large impact parameters. This means that $\vec{q}$ (see Fig. 2) will be perpendicular to the spin alignment axis and, as a result, σ_a will be smaller in magnitude than σ_u. Thus T_{20} should be negative at forward angles. For back-angle reactions, the alignment axis will be nearly parallel to $\vec{q}$ and consequently T_{20} is predicted to be positive. Similar arguments can be used to show that T_{21} and T_{22} should be positive over the entire angular range.

In order to make a more quantitative comparison between the model and the observed tensor analyzing powers, let us consider how the polarized cross section depends on the orientation of the spin alignment axis. The angle between the alignment axis and the incident deuteron beam direction[10] will be denoted by α. According to the model, reactions are most likely to occur when the spin alignment axis points towards the nucleus at the point of transfer. For $Q=0$ the Coulomb trajectories are symmetric about the turning point and consequently the cross section for $Q=0$ should be maximum when α takes on the value

$$\alpha_0=\tfrac{1}{2}(\pi-\theta). \qquad (3)$$

One can use the measured tensor analyzing powers to test this prediction. In order to do this we generalize Eq. (2) to include situations in which the spin alignment axis points in some arbitrary direction in the scattering plane[9]:

$$\sigma_a=\sigma_u[1+2^{-3/2}(3\cos^2\alpha-1)T_{20}+\sqrt{3}\,\sin\alpha\cos\alpha T_{21}+\tfrac{1}{2}\sqrt{3}\,\sin^2\alpha T_{22}]. \qquad (4)$$

Using this formula and measured values of the tensor analyzing powers, one can determine the value of α for which σ_a is largest. In Fig. 3 we compare values of α_0 given by Eq. (3) with those calculated from the tensor analyzing power measurements shown in Fig. 1. The agreement between the measurements and the predictions of the model is quite good.

In the classical model the tensor analyzing powers are nonzero because the deuteron is not spherically symmetric and because the neutron wave function has a nonzero curvature. At first sight, it would seem that this effect is too small to produce substantial analyzing powers. In order to complete our discussion, we will briefly describe how the model can be used to predict the magnitude of the tensor analyzing powers.

We begin with the DWBA expression (assuming a spin-zero target nucleus) for the transition matrix:

$$T\propto\int d^3r_p\,d^3r_d\,\chi_p^{(-)*}(\vec{r}_p)\psi_n^*(\vec{r})\times V_{np}(\vec{\rho})\varphi_d(\vec{\rho})\chi_d^{(+)}(\vec{r}_d). \qquad (5)$$

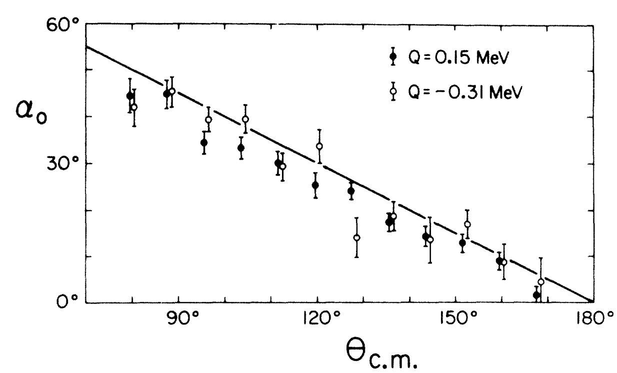

FIG. 3. Values of α_0. The straight line shows the values of α_0 predicted from the classical model for $Q=0$. The data points show the values calculated from the measured tensor analyzing powers.

In Eq. (5) ψ_n is the bound-neutron wave function, χ_p and χ_d are the distorted waves in the proton and deuteron channels, respectively, V_{np} is the neutron-proton potential, and φ_d is the deuteron internal wave function.

For sub-Coulomb reactions the distorted waves in Eq. (5) cause the integrand to peak at the point $\vec{q}$ (Ref. 3). Since the distorted waves play no additional role in the model, we will simplify the integral by replacing χ_p and χ_d with a δ function which requires that the neutron transfer takes place when $\vec{r}_p$ and $\vec{r}_d$ are close to[11] the point $\vec{q}$; i.e.,

$$\chi_p^{(-)*}(\vec{r}_p)\chi_d^{(+)}(\vec{r}_d) \to \delta(\vec{q}-\tfrac{1}{2}(\vec{r}_p+\vec{r}_d)). \quad (6)$$

Equation (6) requires that the average of $\vec{r}_p$ and $\vec{r}_d$ coincide with $\vec{q}$, but since $V_{np}(\vec{\rho})$ is a short-range function, the resulting integrand will be substantial only when both $\vec{r}_p$ and $\vec{r}_d$ are close to $\vec{q}$. The integrand can further be simplified by replacing the neutron wave function by its asymptotic form. For the case in which the neutron orbital angular momentum l_n is zero, we have[12]

$$\psi_n^*(\vec{r}_n) \to \exp(-\kappa r_n)/r_n. \quad (7)$$

With these simplifications, Eq. (5) reduces to

$$T \propto \int d^3\rho\,[V_{np}(\vec{\rho})\varphi_d(\vec{\rho})\exp(-\kappa|\vec{q}+\tfrac{3}{4}\vec{\rho})/|\vec{q}+\tfrac{3}{4}\vec{\rho}|]. \quad (8)$$

The integral in Eq. (8) can be evaluated in closed form provided that we assume $q > 3\rho/4$.[13] With this assumption the integrand can be simplified by making use of Eq. (16.22) of Jackson[14] and the tensor analyzing powers can be calculated using the method of Johnson.[15] The result is that the analyzing powers are given by

$$T_{2m}(\theta) = AY_2^{\,m}(\gamma, 0), \quad (9)$$

where

$$A = -2(\tfrac{4}{5}\pi)^{1/2}(\Delta + 8^{-1/2}\Delta^2)/(1+\Delta^2) \quad (10)$$

and

$$\Delta = [u_2(\tfrac{3}{4}i\kappa)/u_0(\tfrac{3}{4}i\kappa)][1 + 3/\kappa q + 3/(\kappa q)^2]. \quad (11)$$

In Eq. (11) u_0 and u_2 are, respectively, the S-state and D-state parts of the deuteron wave function in momentum space.[15]

The predictions of the classical model which are contained in Eqs. (1) and (9)–(11) can be used to calculate the tensor analyzing powers for $l_n = 0$ sub-Coulomb (d, p) reactions with Q near zero. The results of this calculation[16] for the $\frac{1}{2}^+$ transition on ^{208}Pb at 9 MeV are given by the dashed curves in Fig. 1. The agreement between the classical calculation and the DWBA calculation is surprisingly good in view of the approximations made and the simplicity of the classical result. We conclude that despite its simplicity, the model which we have proposed properly describes the effect of the deuteron D state in sub-Coulomb (d, p) reactions.

*Work supported in part by the U. S. Atomic Energy Commission.

[1]L. D. Knutson, E. J. Stephenson, N. Rohrig, and W. Haeberli, Phys. Rev. Lett. 31, 392 (1973).

[2]The deuterons are said to be aligned, or tensor polarized, when the population of the $m = 0$ magnetic substate is different from $\frac{1}{3}$ for some choice of quantization axis. The analyzing powers are defined according to the Madison convention as given in *Polarization Phenomena in Nuclear Reactions*, edited by H. H. Barschall and W. Haeberli (Univ. of Wisconsin Press, Madison, 1971).

[3]L. J. B. Goldfarb, in *Lectures in Theoretical Physics*, edited by P. D. Kunz, D. A. Lind, and W. E. Brittin (Univ. of Colorado Press, Boulder, 1966), Vol. VIII C, p. 445.

[4]S. E. Vigdor, R. D. Rathmell, and W. Haeberli, Nucl. Phys. A210, 93 (1973).

[5]R. H. Lemmer, Nucl. Phys. 39, 680 (1962); D. Trautmann and K. Alder, Helv. Phys. Acta 43, 363 (1970).

[6]The calculations were carried out using the method of R. C. Johnson and F. D. Santos, Particles Nucl. 2, 285 (1971). The parameters used in the calculations are those given in Ref. 1.

[7]The justification for this assumption can be found in Ref. 4.

[8]One can think of these ellipses as surfaces over which the magnitude of the deuteron wave function remains constant.

[9]S. E. Darden, in *Polarization Phenomena in Nuclear Reactions*, edited by H. H. Barschall and W. Haeberli (Univ. of Wisconsin Press, Madison, 1971), p. 39.

[10]Here we will consider only the case in which the spin axis lies in the scattering plane. The sign of α is chosen so that the solid-line ellipse in Fig. 2 has $\alpha = 45°$ or $-135°$ and the dashed-line ellipse has $\alpha = 135°$ or $-45°$.

[11]It might seem more natural to require that both $\vec{r}_p$ and $\vec{r}_d$ coincide with the point $\vec{q}$. However, this would imply that the neutron-proton separation must be zero. Since the D-state effect depends on the finite size of the deuteron, it is necessary to allow for $\vec{r}_p \neq \vec{r}_d$. The δ function chosen treats the deuteron and proton symmetrically and places no restriction on the neutron-proton separation.

[12]The results which follow are not applicable for $l_n > 0$. For nonzero orbital angular momentum, the neutron wave function depends on the angular coordinates of $\vec{r}_n$ and, as a result, the calculations are significant-

ly more complicated than for $l_n = 0$.

[13]This assumption is valid because q, which is greater than 10 fm for the cases considered here, is much larger than the range of V_{np}.

[14]J. D. Jackson, *Classical Electrodynamics* (Wiley, New York, 1962), p. 541.

[15]R. C. Johnson, Nucl. Phys. A90, 289 (1967).

[16]In the calculation the functions u_2 and u_0 were evaluated using the same approximations and parameters (see Ref. 6) which were used in the DWBA calculations. The values of q were calculated using Eq. (6) of Ref. 4. The kinetic energy was taken to be the average of the center-of-mass energies in the proton and deuteron channels.

Asymptotic Limit of $\sigma(e^+ + e^- \to \text{hadrons})/\sigma(e^+ + e^- \to \mu^+ + \mu^-)$†

Hidezumi Terazawa
The Rockefeller University, New York, New York 10021
(Received 31 October 1973)

Combining the low-energy theorem by Crewther and by Chanowitz and Ellis on the anomalous vertex function for partially conserved dilation current and a new result from the most conventional vector-meson dominance, we predict the asymptotic limit of the ratio $\sigma(e^+ + e^- \to \text{hadrons})/\sigma(e^+ + e^- \to \mu^+ + \mu^-)$ to be $R \cong 16\pi^2/f_0^2 = 5.7 \pm 0.9$, which is in remarkable agreement with the preliminary data from the Cambridge Electron Accelerator e^+-e^- colliding-beam experiment but in strong contradiction with the other predictions $R = \frac{2}{3}$ and $R = 2$ in the single- and three-triplet fractionally charged quark models.

One of the most intriguing experimental discoveries recently reported is the surprisingly large total cross section for hadron production which has been observed in the e^+-e^- colliding-beam experiment at Cambridge Electron Accelerator (CEA).[1] The data have been analyzed in such a way that the ratio $R(q^2)$ of the observed total cross section for $e^+ + e^- \to$ hadrons to the theoretical one ($\simeq 4\pi\alpha^2/3q^2$) for $e^+ + e^- \to \mu^+ + \mu^-$ is calculated to be 4.7 ± 1.1 at $q^2 = 16$ GeV^2 and to be about 6.3 at $q^2 = 25$ GeV^2, where q^2 is the square of the total c.m. energy of the colliding beams. A naive comparison of the CEA data with the Frascati data[2] leads to the conclusion that $R(q^2)$ continues to rise as q^2 increases from above 1 up to 25 GeV^2. On the other hand, many authors[3] expect that $R(q^2)$ approaches a constant when q^2 goes up. It is now extremely interesting to ask what the asymptotic value for $R(q^2)$ is. The purpose of this short note is to report our new prediction for $R = \lim_{q^2 \to \infty} R(q^2)$, if it exists, based on the anomaly in partially conserved dilation current (PCDC) and the usual vector-meson dominance.

Let us start with the definition of the photon form factor for the trace of the stress-energy tensor,

$$\langle \gamma(\epsilon_1, k_1) | \theta_\lambda{}^\lambda(0) | \gamma(\epsilon_2, k_2) \rangle = -(\epsilon_1 \cdot \epsilon_2 k_1 \cdot k_2 - \epsilon_1 \cdot k_2 \epsilon_2 \cdot k_1) F((k_1 - k_2)^2). \quad (1)$$

Recently, Crewther[4] and, independently, Chanowitz and Ellis[5] have pointed out, on the canonical anomaly existing in the PCDC,[6] that the coefficient of the anomalous term is completely determined by R. Furthermore, they have found that, because of the anomaly, $F(0)$ does not vanish and is given by

$$F(0) = -(e^2/12)\int dx\, dy\, x \cdot y \langle 0 | T(J_\mu(x) J^\mu(0) \theta_\lambda{}^\lambda(y)) | 0 \rangle = (e^2/6\pi^2) R. \quad (2)$$

If we assume vector-meson dominance, the left-hand side of Eq. (1) can be approximated by

$$(e^2/f_\rho^2) \langle \rho(\epsilon_1, k_1) | \theta_\lambda{}^\lambda(0) | \rho(\epsilon_2, k_2) \rangle + (\text{isoscalar terms}), \quad (3)$$

where f_ρ is the γ-ρ coupling constant ($f_\rho^2/4\pi = 2.2 \pm 0.3$). Now, the point is that we can evaluate the matrix element in (3) by the definition of the stress-energy tensor when $k_1 = k_2$, namely,

$$\langle \rho(\epsilon_1, k) | \theta_{00}(0) | \rho(\epsilon_2, k) \rangle = -2k_0 k_0 (\epsilon_1 \cdot \epsilon_2 - \epsilon_1 \cdot k \epsilon_2 \cdot k / k^2). \quad (4)$$