New Determination of the Asymptotic *D*-State-to-*S*-State Ratio of the Deuteron

N. L. Rodning and L. D. Knutson
University of Wisconsin, Madison, Wisconsin 53706
(Received 29 July 1986)

The asymptotic D/S ratio of the deuteron, η, is deduced from new measurements of tensor analyzing powers for sub-Coulomb (d,p) reactions. Values of η obtained under various experimental conditions (i.e., corresponding to different analyzing powers, bombarding energies, target nuclei, and final states) are consistent within their statistical uncertainties. A weighted average of the individual measurements gives $\eta = 0.0256 \pm 0.0004$. This result is compared with predictions obtained from a variety of theoretical models.

PACS numbers: 21.40.+d, 13.75.Cs, 24.70.+s, 25.45.Gh

The deuteron D-state probability, P_D, has traditionally held an important place in discussions of the deuteron wave function. However, it has recently become clear[1,2] that P_D is, for fundamental reasons, inaccessible to experiment. On the other hand, the asymptotic D-state-to-S-state wave-function ratio, η, can be determined experimentally, and as a result η is now viewed by many to be the fundamental D-state parameter. In the past few years, the asymptotic D/S ratio has been the subject of a great deal of new work. Considerable progress has been made in the development of experiment methods for the determination of η, while at the same time theorists have made important advances in understanding the relationship between η and other properties of the nucleon-nucleon system, thereby making it possible to exploit the new experimental results.

The purpose of the present Letter is to report the result of a new experimental determination of η which we consider to be significantly more reliable than any previous measurement. The new values of η is derived from polarized-beam measurements for sub-Coulomb (d,p) reactions on ^{136}Xe and ^{208}Pb.

Previous measurements of η have come from two kinds of experiments. The sub-Coulomb (d,p) method[3] has been employed most recently by Stephenson and Haeberli[4] and yields a value of η (see Goddard, Knutson, and Tostevin[5]) which is accurate to $\pm 3\%$ (see Table I). The second method involves analysis of polarized-beam measurements for d-p elastic scattering. It is based on the observation that the neutron-exchange process gives rise to a pole in the scattering amplitude which, for physical values of the bombarding energy, is located at an unphysical scattering angle. Amado, Locher, and Simonius[13] have shown that at the pole position the tensor analyzing powers[14] depend in a simple way on η, and have suggested that η can be determined by extrapolation of measurements in angle to the pole position.

Pole-extrapolation analysis of d-p elastic-scattering data has been the subject of many papers over the past few years,[6,7,15–22] and values of η with errors as small as 2.2% have been reported. However, in much of this work the uncertainties reported for η reflect only the errors in the data, and no attempt is made to determine the systematic errors associated with the extrapolation or with other aspects of the analysis. This omission is a serious one, since several authors[6,21,22] have concluded that extrapolations carried out in the usual manner with a truncated polynomial series give values of η which are not reliable (i.e., in error by 5% or more).

The pole-extrapolation method has also been applied to the reaction $^2H(d,p)^3H$ by Borbely *et al.*,[8] and a value of η with an uncertainty of only 1.5% has been reported. However, in this case again the quoted error includes no contribution from the systematic error in the analysis.

Some representative pole-extrapolation results are given in Table I. Here we also list values of η from some selected nucleon-nucleon potentials,[9,10] as well as the results of two theoretical studies[11,12] in which the value of η is deduced from properties of the two-nucleon system by procedures which rely on the dominance of one-pion exchange (OPE) for the long-range nucleon-nucleon interaction.

In the sub-Coulomb (d,p) method the value of η is determined by comparison of measurements of the tensor analyzing powers with calculations based on the distorted-wave Born approximation (DWBA). This method relies on the idea that for very low bombarding energies the Coulomb repulsion causes the reactions to take place

TABLE I. Values of η from various sources. The quoted uncertainties in η are given in parentheses.

Ref.	Method	η
5	Sub-Coulomb (d,p)	0.0271(8)
6	Pole extrapolation (elastic)	0.0264(14)
7	Pole extrapolation (elastic)	0.0270(6)[a]
8	Pole extrapolation [$^2H(d,p)^3H$]	0.0272(4)[a]
9	Reid soft-core potential	0.0262
10	Paris potential	0.0261
11	Theoretical result[b,c]	0.0259(3)
12	Theoretical result[b]	0.0268(7)

[a]Quoted error includes no contribution from uncertainty in analysis.
[b]Derived from measured deuteron properties plus OPE.
[c]From Eq. (17) of Ref. 11 with $f^2 = 0.0776(9)$ and $\Lambda \to \infty$.

far outside the nuclear surface, and that consequently the analyzing powers are almost totally insensitive to nuclear interactions with the target nucleus. Under the conditions encountered in these reactions, DWBA theory should be sufficiently reliable to permit accurate predictions of the analyzing powers. Furthermore, it is found that the tensor analyzing powers are nearly proportional to the value of η.[23] Additional details are given in Refs. 3, 4, and by Knutson.[24]

The experiment we have done differs from previous sub-Coulomb (d,p) experiments in several important respects. First, we have used lower energies than in the past, which enhances the reliability of the calculations. The present measurements were obtained at $E_d = 6.0$ and 7.0 MeV for ^{208}Pb, and $E_d = 4.5$ and 5.5 MeV for ^{136}Xe. Second, we have made significant improvements in the statistical uncertainty and also in the overall normalization uncertainty of the measurements. Finally, we have obtained measurements under a variety of conditions, making it possible to perform meaningful consistency checks. For each nucleus and bombarding energy we have measured the T_{20} tensor analyzing power for two final states in the residual nucleus.[25] In addition, for ^{208}Pb at 7.0 MeV, we have measurements for both T_{20} and T_{21}. We are thus able to extract ten statistically independent values of η.

All of the measurements were made at the University of Wisconsin tandem accelerator laboratory with deuteron beams produced by a colliding-beam polarized-ion source.[26] Briefly, the tensor-analyzing-power measurements involve determining the relative counting rates for beams with large positive and large negative polarizations.[14] The polarization of the beam was measured with a ^{3}He polarimeter[27] located downstream of the main scattering chamber. As part of the present experiment the polarimeter was recalibrated (for T_{20} measurements) against an absolute polarization standard at the energies used in the main experiment. A complete description of the experimental details will be presented in a later publication.

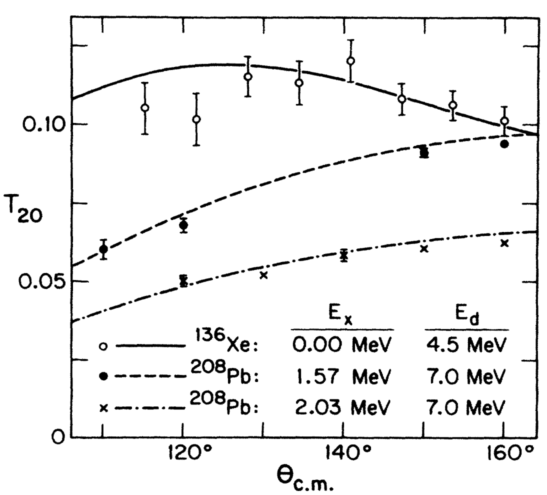

FIG. 1. Angular distributions of T_{20} for three representative cases. The curves show DWBA predictions obtained with the Reid soft-core deuteron wave function ($\eta = 0.0262$). These calculations contain no adjustable parameters.

The measurements of T_{20} for three selected cases are presented in Fig. 1. The error bars shown here represent the statistical uncertainties in the measurements. The curves in Fig. 1 are DWBA calculations, which we obtained by employing the Reid soft-core nucleon-nucleon interaction[9] (for which $\eta = 0.0262$). The calculations are identical in content to those performed by Goddard, Knutson, and Tostevin.[5] In particular, the calculations use finite-range DWBA, include tensor interactions arising from the deuteron quadrupole moment[28] and from the electric polarization of the deuteron by the Coulomb field of the nucleus[29] as well as tensor forces of nuclear origin,[30] and include corrections for p-wave admixtures in the deuteron wave function which arise from deuteron stretching.[31] The optical-model potentials were taken from Becchetti and Greenless[32] and Daehnick, Childs, and Vrcelj.[33] The calculations contain no free parameters.

By making use of the fact that T_{20} and T_{21} are essentially proportional to the value of η used in the calculation, one can easily find the optimum value of η for each angular distribution. The results for the ten separate cases are presented in Table II. Here we have also listed the minimum χ^2/N (chi squared per degree of freedom) for each case. The uncertainty in η is made up of three contributions. The statistical error $\delta\eta_S$ and the normalization error $\delta\eta_N$ (which results from the errors of the polarimeter calibration experiment) can be determined in a straightforward way, and are listed in Table II. The third contribution $\delta\eta_C$ results from the uncertainties in the calculation and is more difficult to estimate.

The calculational uncertainty arises primarily from uncertainties in the optical-model potentials. Briefly, the procedure for the determination of $\delta\eta_C$ involves assigning

TABLE II. Values of η from each angular distributon. Energies (E_d, E_x) are given in MeV and uncertainties $(\delta\eta_S, \delta\eta_N, \delta\eta_C)$ in percent.

Target	$T_{\kappa q}$	E_d	E_x	χ^2/N	η	$\delta\eta_S$	$\delta\eta_N$	$\delta\eta_C$
^{208}Pb	T_{20}	6.0	2.03	1.8	0.0253	1.8	0.7	1.1
			1.57	0.6	0.0261	2.5	0.7	1.1
		7.0	2.03	1.4	0.0254	1.1	0.6	1.5
			1.57	0.3	0.0256	0.9	0.6	1.3
	T_{21}	7.0	2.03	1.3	0.0256	2.4	1.0	1.3
			1.57	2.4	0.0268	3.8	1.0	1.3
^{136}Xe	T_{20}	4.5	0.00	1.1	0.0258	1.9	0.8	1.6
			0.60	0.7	0.0263	1.8	0.8	2.0
		5.5	0.00	1.2	0.0254	1.3	0.9	3.2
			0.60	1.7	0.0253	1.3	0.9	4.5

an uncertainty to each term in the optical-model potentials. For the central and spin-orbit terms the potential depth was assumed to be uncertain by $\pm 30\%$, while for the tensor potentials we have taken the uncertainty to be $\pm 50\%$ of the folding-model value.[30] We then find the contribution to the uncertainty in η by determination of the sensitivity of the calculation to each term. The final error also includes a contribution from the uncertainty in the tensor component of the polarizability potential.[29] Finally, we have included an extra calculational uncertainty, taken to be 1.0% of the calculated analyzing powers, to account for errors which were not specifically investigated (such as relativistic effects, effects of channel coupling, etc.). The values of $\delta\eta_C$ (see Table II) are obtained by addition of the various error contributions in quadrature.

The values of η obtained from the ten individual angular distributions are shown in Fig. 2 along with error bars which represent the statistical uncertainties. The important point to note here is that the individual values of η are consistent to within the statistical errors (i.e., the solid line has $\chi^2/N = 0.68$). We believe that the internal consistency of these results should be taken as strong evidence in support of the validity of the sub-Coulomb (d,p) method.

Our final result for η is obtained by taking a weighted average of the individual determinations with weighting factors chosen so as to minimize the error of the final result. This procedure is somewhat complicated because of correlations among the errors of the individual points. Whereas the statistical errors are completely uncorrelated, we assume that the calculational errors for the ten cases are completely correlated. Correlations among the normalization errors are more complicated, but are nevertheless accounted for in the analysis. The final result is

$$\eta_{\text{expt}} = 0.0256 \pm 0.0004. \quad (1)$$

The value of η which we obtain is somewhat smaller than the best results available from previous experimental work. Although most of the previous determinations have uncertainties which are large enough to overlap or nearly overlap our error bar, there is a marked discrepancy between our result and that obtained by Borbely *et al.*[8] from a pole-extrapolation analysis of ^{2}H$(d,p)^3$H data. In view of this, it is clearly important that efforts be made to determine whether systematic errors associated with the pole-extrapolation analysis (assumed in Ref. 8 to be negligible) make a significant contribution to the uncertainty in η. New ^{2}H$(d,p)^3$H experiments and independent attempts to perform the required extrapolations might also be useful in helping to clarify the nature of the discrepancy.

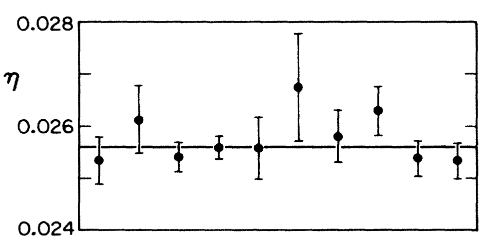

FIG. 2. Values of η obtained under various experimental conditions. The points are plotted in the order in which they appear in Table II. The error bars show the statistical uncertainties, and the solid line has $\chi^2/N = 0.68$.

It is also of interest to compare the measured value of η with the predictions obtained from nucleon-nucleon potential models. Results for a large number of models are given in Refs. 11 and 12. In nearly all cases the predicted value of η falls in the range 0.0255–0.0265. However, if one considers only those models in which the π-N coupling constant is close to the experimental value[34] ($f^2 = 0.0776 \pm 0.0009$), η is generally quite close to 0.0261. The Paris potential[10] (see Table I) falls into this category.

Ericson and Rosa-Clot[11] have made a detailed study of the relationship between f^2 and η. By employment of Green's-function approach they have shown that the D-state component of the wave function is generated by a source function which depends on both the nucleon-nucleon tensor potential, V_T, and the S-state wave function, u. Because the D-wave central potential is strongly repulsive (as a result of the combined effect of the centrifugal barrier and the repulsive OPE potential), the D-state wave function is mainly sensitive to the long-range parts of u and V_T, and it follows that η is primarily determined by the properties of the OPE interaction. These authors conclude that within the context of conventional meson-exchange models, η is essentially proportional to f^2, and that for $f^2 = 0.0776 \pm 0.0009$ one should expect

$$\eta_{\text{th}} = 0.0259 \pm 0.0003. \quad (2)$$

Our experimental result is consistent with the conventional meson-nucleon picture [Eq. (2)]. Nevertheless, it is interesting to note that experimental values of η slightly smaller than Eq. (2) are not completely unexpected. In particular, if one includes a πNN form factor (i.e., if one treats the nucleon as an extended, rather than pointlike, source of pions), significant reductions in η_{th} are obtained.[21] Guichon and Miller[35] have used a quark bag model to investigate not only these form-factor effects, but, in addition, effects which arise from quark antisymmetrization and from explicit interactions between quarks, pions, and gluons. Based on their results, one would expect to obtain a 1% reduction in η (i.e., from 0.0259 to 0.0256) for a quark bag radius of roughly 0.8 fm. The consequences for the deuteron quadrupole moment are predicted to be insignificant for bag radii of this size.[35] It thus appears that the quadrupole moment and our new value of η are both in good agreement with predictions derived from modern quark-based pictures of the nucleon-nucleon interaction.

This work was supported in part by the National Science Foundation. The authors would like to thank Dai Dee Pun Casavant for her assistance in calibrating the polarimeter, and Jeff Tostevin for the computer code used to calculate corrections to the analyzing powers related to deuteron stretching.

[1]R. D. Amado, Phys. Rev. C **19**, 1473 (1979).

[2]J. L. Friar, Phys. Rev. C **20**, 325 (1979).

[3]L. D. Knutson and W. Haeberli, Phys. Rev. Lett. **35**, 558 (1975).

[4]K. Stephenson and W. Haeberli, Phys. Rev. Lett. **45**, 520 (1980).

[5]R. P. Goddard, L. D. Knutson, and J. A. Tostevin, Phys. Lett. **118B**, 241 (1982).

[6]J. T. Londergan, C. E. Price, and E. J. Stephenson, Phys. Lett. **120B**, 270 (1983).

[7]J. Horacek, J. Bok, V. M. Krasnopolskij, and V. I. Kukulin, Phys. Lett. **172B**, 1 (1986).

[8]I. Borbely, V. König, W. Grübler, B. Jenny, and P. A. Schmelzbach, Nucl. Phys. **A351**, 107 (1981); I. Borbely, W. Grüebler, V. König, P. A. Schmelzbach, and B. Jenny, Phys. Lett. **109B**, 262 (1982).

[9]R. V. Reid, Ann. Phys. (N.Y.) **50**, 411 (1968).

[10]M. Lacombe *et al.*, Phys. Rev. C **21**, 861 (1980).

[11]T. E. O. Ericson and M. Rosa-Clot, Nucl. Phys. **A405**, 497 (1983).

[12]S. Klarsfeld, J. Martorell, and D. W. L. Sprung, J. Phys. G **10**, 165 (1984).

[13]R. D. Amado, M. P. Locher, and M. Simonius, Phys. Rev. C **17**, 403 (1978).

[14]W. Haeberli, in *Nuclear Spectroscopy and Reactions,* edited by J. Cerny (Academic, New York, 1974), Pt. A, p. 151.

[15]R. D. Amado *et al.*, Phys. Lett. **79B**, 368 (1978).

[16]H. E. Conzett, F. Hinterberger, P. von Rossen, F. Seiler, and E. J. Stephenson, Phys. Rev. Lett. **43**, 572 (1979).

[17]W. Grüebler, V. König, P. A. Schmelzbach, B. Jenny, and F. Sperisen, Phys. Lett. **92B**, 279 (1980).

[18]F. D. Santos and P. C. Colby, Phys. Lett. **101B**, 291 (1981), and Nucl. Phys. **A367**, 197 (1981).

[19]P. C. Colby, Nucl. Phys. **A370**, 77 (1981).

[20]L. J. Allen and M. W. Kermode, Phys. Lett. **109B** 259 (1982).

[21]G. H. Berthold and H. Zankel, Phys. Rev. C **30**, 14 (1984).

[22]D. D. Pun Casavant, J. G. Sowinski, and L. D. Knutson, Phys. Rev. Lett. **154B**, 6 (1985).

[23]This contention has recently been challenged by M. W. Kermode, L. J. Allen, and J. P. McTavish, J. Phys. G **8**, L71 (1982). These authors assume that the tensor analyzing powers scale with the local-energy parameter D_2, and show that one can make substantial changes in D_2 without changing η. However, detailed calculations show that, to a good approximation, the analyzing powers actually scale with η, not D_2. Further details will be presented in a subsequent publication.

[24]L. D. Knutson, Ann. Phys. (N.Y.) **106**, 1 (1977).

[25]Data were obtained for states at $E_x = 1.57$ and 2.03 MeV in ^{209}Pb and at $E_x = 0.00$ and 0.60 MeV in ^{137}Xe.

[26]W. Haeberli *et al.*, Nucl. Instrum. Methods **196**, 319 (1982).

[27]K. Stephenson and W. Haeberli, Nucl. Instrum. Methods **169**, 483 (1980).

[28]J. E. Kammeraad and L. D. Knutson, Nucl. Phys. **A435**, 502 (1985).

[29]M. H. Lopes, J. A. Tostevin, and R. C. Johnson, Phys. Rev. C **28**, 1779 (1983).

[30]The folding-model tensor potentials of P. W. Keaton and D. D. Armstrong, Phys. Rev. C **8**, 1692 (1973), were used, except that the strength of the potential was scaled by a factor of 0.5 to reproduce measurements of tensor analyzing powers in elastic scattering of deuterons on ^{208}Pb and ^{136}Xe.

[31]J. A. Tostevin and R. C. Johnson, Phys. Lett. **85B**, 14 (1979).

[32]F. D. Becchetti and G. W. Greenless, Phys. Rev. **182**, 1190 (1969).

[33]W. W. Daehnick, J. D. Childs, and Z. Vrcelj, Phys. Rev. C **21**, 2253 (1980).

[34]See Ref. 11, footnote to Eq. (A.16).

[35]P. A. M. Guichon and G. A. Miller, Phys. Lett. **134B**, 15 (1984).