Determination of the Phase Shifts for p-d Elastic Scattering at $E_p = 3$ MeV

L. D. Knutson, L. O. Lamm,* and J. E. McAninch
Physics Department, University of Wisconsin, Madison, Wisconsin 53706
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Phase shift parameters for p-d elastic scattering are extracted from new measurements of spin observables at $E_p = 3$ MeV. The experimental phase shifts are compared with predictions obtained from Faddeev calculations. The agreement between theory and experiment is generally good. However, we find that there are significant discrepancies for the ${}^2S_{\frac{1}{2}}$ phase shift and for the S-D mixing parameters.

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There is currently a great deal of interest in understanding the nuclear three-body system. Over the last several years a substantial amount of progress has been made in the development of the theoretical means for addressing the three-body problem (for example, techniques for solving the Faddeev equations). It is now well established (see, for example, Ref. [1]) that conventional two-body NN forces are not sufficiently strong to reproduce the observed ^{3}H and ^{3}He binding energies, and the role of three-body forces in resolving this discrepancy has been studied extensively [2–4].

Experimental results for n-d and p-d elastic scattering also have the potential to provide new information concerning the nature of the fundamental interactions in complex nuclear systems. As the theoretical capabilities continue to advance we can anticipate that the scattering data will begin to take on added importance. It has been known for some time that measurements of the nucleon analyzing power, A_y, for N-d scattering are very sensitive to the NN ${}^3P_j$ partial waves [5], and this sensitivity has recently been exploited in an effort to constrain the NN ${}^3P_j$ forces [6]. It has also been suggested [5–7] that comparison of n-d and p-d results may provide information concerning possible charge independence and/or charge symmetry violation, and one can expect that N-d measurements will ultimately play an important role in understanding the nature and role of nuclear three-body forces.

Our purpose in the present Letter is to present new measurements of spin observables for p-d elastic scattering. The measurements will be compared with existing state-of-the-art Faddeev calculations [8]. These calculations provide a good qualitative description of the data, but at the quantitative level there are discrepancies. The nature of the differences between the measured and calculated polarization observables will be studied by comparing phase shifts extracted from the data with the calculated phase shifts. Comparison of theory and experiment at the level of the phase shifts is clearly of value, since it is often possible to relate individual splittings or mixing parameters with specific components of the NN force. However, comparisons of this kind have not been fruitful in the past since the measurements have not been of sufficient quality to allow reliable and accurate determination of the phase shift parameters.

We have obtained new measurements of the polarization observables for p-d elastic scattering at $E_p = 3$ MeV and at the corresponding deuteron energy, $E_d = 6$ MeV. The new data set consists of measurements of the proton analyzing power, A_y, the deuteron vector analyzing power, iT_{11}, and the three deuteron tensor analyzing powers, T_{20}, T_{21}, and T_{22}. The measurements are accurate to typically ± 0.0004. The bombarding energy is below the threshold for deuteron breakup, which simplifies the extraction of the phase shift parameters.

The measurements were carried out at the University of Wisconsin tandem accelerator laboratory. Proton and deuteron beams from the crossed-beam polarized ion source [9] were accelerated and momentum analyzed with a 90° bending magnet. The scattering measurements were carried out in a 1 m diameter scattering chamber which was filled with either hydrogen or deuterium gas at a pressure of 200 Torr. The details of the experimental arrangement are similar to those described in Ref. [10]. Protons and deuterons from elastic scattering were detected with silicon surface barrier detectors located approximately 25 cm from the center of the chamber. The polarization of the beam was monitored with a polarimeter [11,12] located just downstream of the main scattering chamber. To minimize systematic errors the polarization was reversed at intervals of typically 250 ms.

The observed spectra were generally very clean with well isolated peaks. In most cases the backgrounds were less than 1%, and background subtraction usually changed the measured analyzing powers by less than 0.0001. For situations in which the shift from background subtraction was an appreciable fraction of the statistical error, an estimate of the background error was included in the final uncertainty.

The measurements were taken in a series of 2 h runs, and as a general rule many such runs (typically from 5 to 30) were required to achieve the desired level of statistical accuracy. The repeated measurements were checked

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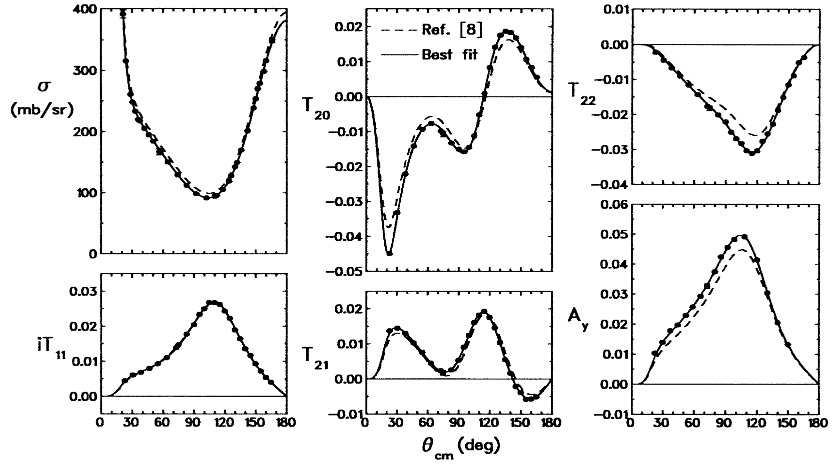

FIG. 1. Measurements of the differential cross section and polarization observables for p-d elastic scattering at $E_p = 3$ MeV. The experimental uncertainties are generally about the size of the plotting symbols. The dashed curves show the predictions obtained from the Faddeev calculations of Ref. [8]. The solid curves are a phase shift fit. The two curves are indistinguishable for iT_{11}. The cross section measurements are from Ref. [14].

for consistency, and in those cases where the scatter was larger than expected from statistical fluctuations alone (specifically whenever the confidence level was less than 10%), the statistical error was increased by the square root of χ^2 to allow for possible added uncertainties. The procedure used to extract the deuteron analyzing powers from the observed counting rates is described in Ref. [10]. For the proton analyzing power we use the usual cross ratio method [13].

The measured analyzing powers are shown in Fig. 1 along with differential cross section data taken from Ref. [14]. The uncertainties, which are not shown in the figure, are generally about the size of the plotting symbols. The overall normalization uncertainties are estimated to be 1% for T_{20}, 1.5% for A_y, and 2% for the remaining analyzing powers. Tables of the data are available from the authors on request. We note in passing that the present measurements are in good agreement with the high-quality unpublished data of Sagara [15].

The dashed curves in Fig. 1 show the predictions obtained from the Faddeev calculations of Berthold, Stadler, and Zankel [8]. These calculations employ the PEST 16 potential [16], which is a rank-one separable interaction with parameters chosen to approximate the Paris potential [17]. The calculation includes interactions in the $^1S_0$, $^3S_1$-$^3D_1$, $^1P_1$, $^3P_0$, $^3P_1$, and $^3P_2$ NN states. Like the Paris potential itself, the PEST 16 potential underbinds the ^{3}H and ^{3}He nuclei by roughly 1 MeV [18].

The calculations of Ref. [8] employ certain approximations in the treatment of the Coulomb interaction. The approximations include replacing the Coulomb t matrix in the Faddeev equations by the potential itself, and neglecting admixtures of isospin $\frac{3}{2}$ states. The actual calculations are carried out with a screened Coulomb potential, but a renormalization procedure is employed to correct for the long range part of the Coulomb interaction, and the unscreened Coulomb p-d t matrix is determined by taking the limit of the screened Coulomb results for a series of increasing values of the screening radius.

Overall, the calculations provide a reasonably good qualitative description of the data. For the differential cross section the calculation is too large by about 5% for angles near 90°. The deuteron vector analyzing power and the T_{21} tensor analyzing power are both well reproduced by the calculation, while for the proton analyzing power and for T_{20} and T_{22} the calculations tend to be too small in magnitude, typically by 10% or so.

While the Faddeev calculation shown reproduces the qualitative features of the measurements, the fit is not good in a quantitative sense. Overall, the reduced χ^2 is greater than 80 per data point. We find that the χ^2 can be improved dramatically by allowing small changes in a few of the phase shift parameters. For example, if we permit the $^2S_{\frac{1}{2}}$ phase shift, the $^2S_{\frac{1}{2}}$-$^4D_{\frac{1}{2}}$ mixing parameter, and the $^2P_{\frac{3}{2}}$-$^4P_{\frac{3}{2}}$ mixing parameter to vary, the reduced χ^2 improves by more than a factor of 5.

In order to explore in greater detail the nature of the discrepancies between the measurements and calculations it is useful to look more closely at the phase shifts. Ideally one would like to determine all of the phase shift parameters from the data and then compare these phase shifts with the theoretical ones. One difficulty is that the phase shifts go to zero only gradually with increasing orbital angular momentum. As a result, the number of free parameters is large and consequently the individual parameters are not well determined.

The solution to this problem is to find a simple but reliable model to calculate the phase shifts for the high partial waves and treat only the low-l phase shifts as adjustable parameters. The model we use is based on the assumption (see Ref. [19]) that the high partial waves are dominated by the neutron exchange process. The procedure we have used is to calculate the phase shift parameters for this process using the plane-wave Born approximation [19]. These calculated phase shifts are then used for the high partial waves. For the particular fit we have chosen to present here, the adjusted parameters consist of the S- and P-wave phase shifts, the D-wave phases (except for the ^2D splitting and one particular combination of ^4D phase shifts that is poorly determined by the data), plus all of the S-D, P-P, P-F, and D-D mixing parameters. We also adjust the overall average ^4F phase shift and one combination of ^4F phases. This particular fit has 24 free parameters, and reproduces the data set (which includes a total of 153 measurements) with a reduced χ^2 of 1.46. The fit is shown by the solid curves in Fig. 1.

For the phase shift fits we have adopted the notation and the sign conventions of Seyler [20]. In particular, the phase shift code we use is based on Eqs. (4) and (41)–(47) of Ref. [20]. To transform between channel-spin and particle-spin representation we use Clebsch-Gordan coefficients with the coupling order $\mathbf{S} = \mathbf{S}_p + \mathbf{S}_d$.

The statistical uncertainty in the phase shift parameters was determined in the usual way from the diagonal elements of the error matrix. For a given parameter this error corresponds closely to the shift required to increase the overall χ^2 of the fit by 1, where all the remaining parameters are varied to minimize χ^2. In addition to the statistical errors, the parameters are subject to systematic uncertainties which can arise from systematic errors in the data set (e.g., beam energy errors, normalization uncertainties, etc.) or from the truncation of the phase shift expansion. To get some idea of how large these errors might be, we carried out a series of fits in which new phase shift parameters were allowed to vary or in which the normalizations of the data sets were floated. From the variation in the parameter values, we conclude that the systematic errors are typically 2–3 times the statistical error.

The experimentally determined phase shift parameters are compared with the theoretical results obtained from the Faddeev calculation of Ref. [8] in Table I. The uncertainties quoted for the experimental phase shifts correspond to the statistical errors only.

The quality of the agreement between the experimental phase shifts and the calculated ones is striking. For the most part, the phase shift parameters differ by only a few tenths of a degree. The largest discrepancy, by far, is for the $^2S_{\frac{1}{2}}$ phase shift. This discrepancy is undoubtedly related to the fact that the potential employed in the calculation does not reproduce the observed ^{3}He binding energy. As a result, it is not surprising that the S-wave phase shift for the $j^\pi = \frac{1}{2}^+$ angular momentum state should be in error. (While the result is not surprising, we know of no one who anticipated it.) It is interesting to note that there is no analogous problem for the $^4S_{\frac{3}{2}}$ state. In this case the calculated phase shift and the experimental one differ by less than 1°.

There are also significant differences between the experimental and theoretical values of the S-D mixing parameters, particularly the $\eta(\frac{1}{2}^+)$ parameter. These parameters, which are essential for obtaining a good fit to the tensor analyzing power data, are presumably sensitive to the NN tensor force and in particular to the mixing of the $^3S_1$ and $^3D_1$ NN states (see Ref. [8]). It is possible that the observed discrepancies are a result of the known deficiencies [8] of the $^3S_1$-$^3D_1$ mixing in the PEST 16 potential. It would be interesting to determine whether this particular problem still remains in calculations with more realistic representations of the NN ten-

TABLE I. Phase shift parameters for p-d elastic scattering at $E_p = 3$ MeV. The calculated values (from Ref. [8]) are based on a solution of the Faddeev equations for the PEST 16 potential. The numerical values quoted are updated results obtained directly from the authors of Ref. [8]. All parameter values are given in degrees.

Parameter	Calculation	Fit
S-wave phases:		
$^4S_{3/2}$	−63.70	-63.95 ± 0.28
$^2S_{1/2}$	−35.69	-24.87 ± 0.35
P-wave phases:		
^4P_j average	23.09	23.37 ± 0.11
$^4P_{5/2}$-$^4P_{3/2}$	−0.06	0.01 ± 0.06
$^4P_{3/2}$-$^4P_{1/2}$	2.68	2.49 ± 0.11
^2P_j average	−7.45	-7.11 ± 0.24
$^2P_{3/2}$-$^2P_{1/2}$	0.22	-0.14 ± 0.25
D-wave phases:		
^4D_j average	−3.95	-3.74 ± 0.09
$^4D_{7/2}$-$^4D_{5/2}$	0.40	0.31 ± 0.03
$^4D_{5/2}$-$^4D_{3/2}$	−0.34	-0.28 ± 0.03
$^4D_{3/2}$-$^4D_{1/2}$	−0.29	-0.16 ± 0.02
^2D_j average	1.99	1.99 ± 0.09
$^2D_{5/2}$-$^2D_{3/2}$	−0.03	0.0 (fixed)
S-D mixing parameters:		
$\eta(\frac{1}{2}^+)$ $[^2S_{1/2} \leftrightarrow {}^4D_{1/2}]$	0.47	2.00 ± 0.10
$\zeta(\frac{3}{2}^+)$ $[^4S_{3/2} \leftrightarrow {}^4D_{3/2}]$	−1.50	-1.20 ± 0.04
$\eta(\frac{3}{2}^+)$ $[^4S_{3/2} \leftrightarrow {}^2D_{3/2}]$	−0.43	-0.28 ± 0.03
P-P mixing parameters:		
$\epsilon(\frac{1}{2}^-)$ $[^2P_{1/2} \leftrightarrow {}^4P_{1/2}]$	−5.26	-5.73 ± 0.13
$\epsilon(\frac{3}{2}^-)$ $[^2P_{3/2} \leftrightarrow {}^4P_{3/2}]$	2.09	2.47 ± 0.05
P-F mixing parameters:		
$\zeta(\frac{3}{2}^-)$ $[^4P_{3/2} \leftrightarrow {}^4F_{3/2}]$	0.40	1.25 ± 0.54
$\eta(\frac{3}{2}^-)$ $[^2P_{3/2} \leftrightarrow {}^4F_{3/2}]$	−3.02	-3.13 ± 0.23
$\zeta(\frac{5}{2}^-)$ $[^4P_{5/2} \leftrightarrow {}^4F_{5/2}]$	−1.07	-0.62 ± 0.20
$\eta(\frac{5}{2}^-)$ $[^4P_{5/2} \leftrightarrow {}^2F_{5/2}]$	−0.39	-0.45 ± 0.04
D-D mixing parameters:		
$\epsilon(\frac{3}{2}^+)$ $[^2D_{3/2} \leftrightarrow {}^4D_{3/2}]$	−0.58	-2.15 ± 0.24
$\epsilon(\frac{5}{2}^+)$ $[^2D_{5/2} \leftrightarrow {}^4D_{5/2}]$	0.26	0.75 ± 0.12

sor interaction.

We believe that the differences between the experimental and theoretical P-P mixing parameters are also physically significant. Although these differences are relatively small, these parameters are well determined by the data and the observed discrepancies are a common feature of all the fits we have done.

For the D-D mixing parameters we also find a relatively large discrepancy between theory and experiment. However, in this case, as one includes more free parameters in the fit, the statistical errors increase and the parameter values are not stable. Consequently, we conclude that this discrepancy is not physically meaningful.

For the remaining parameters, the differences between the experimental and calculated phase shift parameters are generally only a few tenths of a degree. These small differences should be regarded as "fine tuning" of the phase shift parameters. Generally speaking, the shifts are necessary since without them the fit to the data is significantly worse. However, the observed differences do not indicate the existence of any serious deficiency in the calculations. On the contrary, the overall quality of the agreement between theory and experiment is remarkable.

In summary, we find that the Faddeev calculations of Ref. [8] do a good job of reproducing the qualitative features of the new high-precision analyzing power measurements at $E_p = 3$ MeV. The quantitative discrepancies between the measurements and calculations are associated primarily with the $^2S_{\frac{1}{2}}$ phase shift and the S-D mixing parameters. The mixing parameter discrepancy may well be attributable to the inadequate description of the mixing between the $^3S_1$ and $^3D_1$ NN states in the PEST 16 interaction, while the problem with the $^2S_{\frac{1}{2}}$ phase shift is undoubtedly associated with the ^{3}He binding energy discrepancy.

It is important to realize that the $^2S_{\frac{1}{2}}$ phase shift influences not only the differential cross section, but the polarization observables as well. We would suggest that whenever one intends to use polarization measurements to investigate subtle effects in the three-nucleon system (three-body forces, inadequacies of the NN potentials, etc.), it is important to ensure that the interesting effects are not obscured by errors in the calculated $^2S_{\frac{1}{2}}$ phase shift. This could be done, for example, by restricting the calculations to potentials that reproduce the ^{3}He binding energy, or by adjusting the calculated phase shift to account for the known discrepancy.

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* Present address: Physics Department, Presbyterian College, Clinton, SC 29325.

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