Polarization Observables for the *p-d* Breakup Reaction and the Nuclear Three-Body Force

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It is shown that nuclear three-body potentials involve spin operators of a type not allowed for ordinary two-body *NN* interactions. The effect of these operators on the wave function for proton-induced deuteron breakup is discussed, and a class of polarization observables is identified that may have enhanced sensitivity to the three-body forces.

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The nature of the nuclear three-body force and the role of three-body forces in bound states and scattering problems are subjects of central importance in nuclear physics. It is well understood that three-body forces must be present at some level in nuclear systems. For many simple systems encountered in nature (for example, collections of point charges), the interaction between any pair of particles is unaffected by the presence of a third particle, and therefore the potential energy of a three-body system is just the sum of the pairwise two-body potentials. The situation is more complex for particles that have internal structure. In this case particles can "polarize" each other (alter the internal structure by mixing with excited states), and in this way the presence of a third particle can affect the force between 1 and 2. Since nucleons have internal structure, one should expect that nuclear three-body systems will be governed by a potential that contains three-body terms in addition to the usual pairwise *NN* potentials.

Most of what we currently know about the three-nucleon (3*N*) force comes from theoretical models. Several groups (see, for example, Refs. [1,2]) have constructed three-body force models involving two-pion exchange and Δ excitation which lead to explicit expressions for the 3*N* force. On the experimental side there is some evidence to support the existence of a three-body force of this general nature. It is now well established (see, for example, Ref. [3]) that most "realistic" *NN* potentials are not sufficiently strong to reproduce the observed ^{3}H and ^{3}He binding energies. The predicted binding energies are generally about 7.5 MeV for ^{3}H, well short of the observed 8.48 MeV. If one includes the 2π-3*N* potential, the calculated binding energy increases by roughly 1.5 MeV (see Ref. [4]). Although the final result is still not quite correct, it is noteworthy that the model potentials produce energy shifts of the correct sign and roughly the right order of magnitude.

The effect of 3*N* forces has been studied for a variety of observable quantities, the ^{3}H binding energy being the prime example. Without attempting to review the field as a whole, one can say that there is a nontrivial amount of circumstantial evidence to support the existence of 3*N* forces. However, we have little or no hard experimental information concerning the nature of the interaction.

It is clear that progress in understanding the 3*N* force depends critically on our ability to identify observables that are sensitive to various aspects of the potential. With this in mind, our goal in this Letter is to describe a class of observables which we believe may be particularly sensitive to 3*N* forces.

We begin by asking what features 3*N* potentials may have that distinguish them from ordinary pairwise *NN* potentials. A seemingly trivial point is that description of a three-body system requires the use of two independent internal coordinates. If we adopt the Jacobi coordinates of Fig. 1, we see that while the pairwise two-body potentials depend only on a single coordinate, ($\mathbf{x}$, $\mathbf{y}+\frac{1}{2}\mathbf{x}$, or $\mathbf{y}-\frac{1}{2}\mathbf{x}$), the 3*N* potential may contain terms that depend simultaneously on both $\mathbf{x}$ and $\mathbf{y}$.

For example, one finds (see Ref. [4]) that the 2π-3*N* potential contains terms of the form

$$Q = (\boldsymbol{\sigma}_2 \cdot \mathbf{x})(\boldsymbol{\sigma}_3 \cdot \mathbf{y}) - (\boldsymbol{\sigma}_2 \cdot \mathbf{y})(\boldsymbol{\sigma}_3 \cdot \mathbf{x}). \quad (1)$$

It is easy to show using ordinary vector algebra that this operator can be rewritten in the form

$$Q = (\boldsymbol{\sigma}_2 \times \boldsymbol{\sigma}_3) \cdot (\mathbf{x} \times \mathbf{y}). \quad (2)$$

The unique feature of this operator is that it involves the coupling of a spin operator with an axial vector constructed from spatial coordinates. The axial-vector nature gives this operator a distinctive behavior under combined rotations and reflections of the spatial coordinates. It is this special feature of the 3*N* potential that we shall exploit.

To generalize the discussion, it is useful to rewrite Eq. (2) in terms of spherical tensors. Since the operator is a dot product of spin and space vectors, we can write

$$Q = \sum_m (-)^m \Lambda_1^m R_1^{-m}, \quad (3)$$

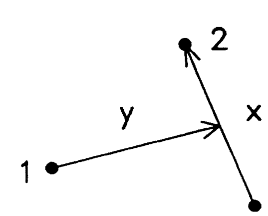

FIG. 1. Jacobi coordinates for the three-body system.

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where Λ is a rank-1 spherical tensor spin operator and R is a rank-1 space operator. One can then easily demonstrate that R is just proportional to the $B_{11,1}$ "bipolar harmonic,"

$$B^{m}_{l_x l_y,L}(\hat{\mathbf{x}},\hat{\mathbf{y}}) \equiv \sum_{\lambda_x \lambda_y} \langle l_x \lambda_x, l_y \lambda_y | Lm \rangle Y^{\lambda_x}_{l_x}(\hat{\mathbf{x}}) Y^{\lambda_y}_{l_y}(\hat{\mathbf{y}}). \quad (4)$$

We shall refer to the operator of Eq. (1) as a rank-1 interaction, since it can be written as a contraction of rank-1 spin and space operators.

In addition to the operator Q discussed above, the 2π-$3N$ potential involves a number of other complex space and spin operators which, when expressed in terms of spherical tensors, are seen to be (at least partially) of rank 1. The common feature of the rank-1 interactions is that they transform as vectors under rotations of the space coordinates. On the other hand, the $3N$ interactions conserve parity, and, since the spin operators do not change sign under parity inversion, the space tensors must also be even under the parity operation. In other words, the spatial operators are all axial vector in nature, by which we mean that they can be expressed in terms of bipolar harmonics with $L = 1$ and with $l_x + l_y$ even.

It should be clear that operators of this class are not permitted for two-body potentials, for the simple reason that one cannot construct a spatial axial vector from a single coordinate.

Let us now discuss the effect that potentials of this kind can have on the wave function of a three-nucleon system. For this discussion we adopt the L-S coupling scheme [5] in which the basis states are of the form

$$|\alpha\rangle = |[(l_x, l_y)L; (s_1, S_x)S]J\rangle. \quad (5)$$

Here l_x and l_y are the angular momenta associated with the coordinates $\mathbf{x}$ and $\mathbf{y}$, respectively, L is the vector sum of l_x and l_y, S_x is the vector sum of s_2 and s_3, S is the total spin angular momentum ($s_1 + S_x$), and J is the sum of L and S. For simplicity we have dropped the isospin quantum numbers. In our discussions we consider only systems of isospin $T_3 = \pm\frac{1}{2}$, with particles 2 and 3 the identical nucleons.

The simplest example to consider is the triton bound state. In this case the wave function consists of a dominant S state ($P_S \simeq 89\%$) together with a small S'-state admixture ($P_{S'} \simeq 1.4\%$), various D-state components ($P_D \simeq 9\%$), and weak P states ($P_P \simeq 0.1\%$). In terms of the L-S coupling notation, the S state includes components with $(l_x, l_y)L = (l, l)0$ and $S_x = 0$, while the S' state corresponds to $L = 0$ and $S_x = 1$. The P states (D states) are characterized by $L = 1$ ($L = 2$), while the requirement of positive parity means that all states must have $l_x + l_y$ even.

In view of the similarity between the angular momentum coupling of Eq. (4) and the coupling implied by Eq. (5), it should be clear that one effect of the axial-vector interactions is to couple the dominant states of the system [for example, the (0,0)0 S state] directly to the P states [for example, (1,1)1]. Therefore, one should expect the $3N$ potentials to have a relatively large effect on the P states, a result which is supported by detailed Faddeev calculations [6].

Let us now turn to the continuum states and specifically to the process of proton-induced deuteron breakup. Here again we use the basis states of Eq. (5), and in this case it is useful to distinguish states of "natural parity," which have $l_x + l_y + L$ even, from states of "unnatural parity," $l_x + l_y + L$ odd. For p-d breakup the initial state of the system is composed primarily of natural parity components [7]. Now the NN tensor force can, in principle, couple these natural parity states to breakup states that have unnatural parity. This happens in the triton as well, where the tensor force gives rise to a small P-state admixture even in the absence of $3N$ forces. In general, however, one expects that the ordinary NN potentials (and this is strictly true for the central and spin-orbit terms) will couple primarily to breakup configurations with natural parity. In contrast, the "axial-vector" components of the $3N$ potential should couple most strongly to states of unnatural parity.

In view of this result, it seems possible that one could enhance the sensitivity of p-d breakup measurements to three-body forces by identifying observables that are especially sensitive to unnatural parity states. As we shall see below, there is a class of polarization observables that may well have this property.

The classification of polarization observables for breakup reactions is somewhat complex. It is well known that for reactions with two-body final states, parity conservation imposes certain restrictions. For example, for reactions initiated with polarized spin-$\frac{1}{2}$ particles, only the component of the polarization normal to the scattering plane affects the scattering cross section. The general theorem is expressed most easily in terms of observables defined using spherical tensor spin operators [8]. Adopting the conventional coordinate frame in which $\hat{\mathbf{z}}$ is along the incident beam direction ($\mathbf{k}_i$) and $\hat{\mathbf{y}}$ is along $\mathbf{k}_i \times \mathbf{k}_f$, one finds that if parity is conserved the reaction analyzing powers satisfy

$$T_{kq} = (-)^k T^*_{kq}. \quad (6)$$

If we now note that A_z (the longitudinal analyzing power) is proportional to T_{10}, that A_x is proportional to $\mathrm{Re}(T_{11})$, and that A_y is proportional to $\mathrm{Im}(T_{11})$, we see that only A_y can be nonzero.

The generalization of the parity theorem to the case of three-body final states is discussed in Ref. [9]. For two-body final states the parity theorem results from a symmetry property of the helicity amplitudes that can be established by considering the effect of an operator $Pe^{i\pi J_y}$ consisting of a rotation of 180° about the y axis followed by a parity inversion. This operation returns all momentum vectors to their original values but reverses the helicities. Assuming that the S matrix is invariant under reflections and rotations one obtains the relationship

$$f_\nu(\mathbf{k}_i;\mathbf{k}_f) = (-)^{s-\nu}\eta f_{-\nu}(\mathbf{k}_i;\mathbf{k}_f), \qquad (7)$$

where ν represents the helicity and η is the product of the intrinsic parities. [If more than one particle has nonzero spin, there are multiple subscripts and $(-)^{s-\nu}$ factors.] The parity theorem follows directly from this result.

For the case of a three-body final state, description of the final-state kinematics requires the use of two relative momenta; for example, the momenta $\mathbf{k}_x$ and $\mathbf{k}_y$ conjugate to the coordinates of Fig. 1. Since the scattering problem now involves three momentum vectors $(\mathbf{k}_i, \mathbf{k}_x, \mathbf{k}_y)$, one can have reaction kinematics for which there is no "reaction plane" that contains all the momenta. Consequently, there is no choice of axes for which the operator $Pe^{i\pi J_y}$ returns all momenta to their original values, and so one obtains a relationship of the form [9]

$$f_\nu(\mathbf{k}_i;\mathbf{k}_x,\mathbf{k}_y) = (-)^{s-\nu}\eta f_{-\nu}(\mathbf{k}_i;\mathbf{k}_x',\mathbf{k}_y'), \qquad (8)$$

where $\mathbf{k}_x'$ and $\mathbf{k}_y'$ are the reflections of $\mathbf{k}_x$ and $\mathbf{k}_y$ in the x-z plane. The parity theorem for three-body final states then becomes

$$T_{kq}(\mathbf{k}_i;\mathbf{k}_x,\mathbf{k}_y) = (-)^k T_{kq}^*(\mathbf{k}_i;\mathbf{k}_x',\mathbf{k}_y'). \qquad (9)$$

The new feature here is that, for noncoplanar reactions, none of the polarization observables are required to be zero. Instead, we may divide the observables into two classes. "Ordinary" observables, of which A_y is the simplest example, have the property of being symmetric under reflections of the final-state momentum vectors in the x-z plane:

$$A_y(\mathbf{k}_i;\mathbf{k}_x,\mathbf{k}_y) = A_y(\mathbf{k}_i;\mathbf{k}_x',\mathbf{k}_y'). \qquad (10)$$

The remaining observables (namely the ones that are required to be zero for two-body final states) are antisymmetric under reflections in the x-z plane. Thus, for example, for the longitudinal analyzing power we now have

$$A_z(\mathbf{k}_i;\mathbf{k}_x,\mathbf{k}_y) = -A_z(\mathbf{k}_i;\mathbf{k}_x',\mathbf{k}_y'). \qquad (11)$$

We shall refer to these quantities as "axial" observables, since they have opposite rotation or reflection behavior compared to the ordinary observables.

At this point one is led to speculate that there could be a connection between these new axial polarization observables and the axial-vector operators which are present in the nuclear three-body potential. Recall that in our discussion of the $3N$ force we saw that the existence of two independent coordinates makes it possible to construct a new class of operators that have a distinctive (axial-vector) behavior under combined rotations and reflections of the spatial coordinates. Similarly, in a three-body final state we now see that the presence of two independent momentum vectors makes possible the existence of a new class of polarization observables that again have unusual behavior under combined rotations and reflections.

To explore this connection in detail we need to construct an explicit expression for the three-body scattering amplitude. To do this we focus on the three-body wave function Ψ which we imagine to be expanded in terms of the states $|\alpha\rangle$ of Eq. (5). To find the scattering amplitude for a particular $\mathbf{k}_x$ and $\mathbf{k}_y$, we need to look at the outgoing-wave part of Ψ in the asymptotic region ($\mathbf{x}$ and $\mathbf{y}$ both large). Furthermore, what is relevant is the value of Ψ at coordinate points $\mathbf{x} \parallel \mathbf{k}_x$ and $\mathbf{y} \parallel \mathbf{k}_y$. What this means is that the presence of a term $|\alpha\rangle$ in the wave function will give rise to a term in the scattering amplitude of essentially the same form but with $\hat{\mathbf{x}}$ replaced by $\hat{\mathbf{k}}_x$ and with $\hat{\mathbf{y}}$ replaced by $\hat{\mathbf{k}}_y$. Explicitly, the scattering amplitude will be of the form [10]

$$F_{s\mu}^{\mu_1\mu_2\mu_3}(\mathbf{k}_i;\mathbf{k}_x,\mathbf{k}_y) \propto \sum_{\alpha lJ\lambda_x\lambda_y\Lambda\mu_x\sigma} (2l+1)^{\frac{1}{2}}\langle l0, s\mu|J\mu\rangle U_{\alpha,sl}^J \langle l_x\lambda_x, l_y\lambda_y|L\Lambda\rangle\langle s_2\mu_2, s_3\mu_3|S_x\mu_x\rangle$$
$$\times \langle s_1\mu_1, S_x\mu_x|S\sigma\rangle\langle L\Lambda, S\sigma|J\mu\rangle Y_{l_x}^{\lambda_x}(\hat{\mathbf{k}}_x) Y_{l_y}^{\lambda_y}(\hat{\mathbf{k}}_y). \qquad (12)$$

In this expression l, s, and J specify the initial angular momentum state of the system, and $U_{\alpha,sl}^J$ is a "collision matrix" that describes the coupling of initial state l, s, J to final state α. The amplitude F is labeled by the channel spin s ($\mathbf{s} = \mathbf{s}_p + \mathbf{s}_d$) and its z component μ, and for the final state by the spin projections, μ_1, μ_2, and μ_3, of the three nucleons.

We may now ask how the presence of various final states α affects the individual polarization observables. We particularly want to focus on the axial polarization observables and on the role of states with unnatural parity, since we believe these states may be strongly influenced by $3N$ forces. Since the observables are quadratic in the amplitudes, each term in the expression for any given observable will involve two interfering states α and α'. The result we find is that the axial observables can be nonzero even in situations where no unnatural parity states are present. However, the angular momentum coupling is such that terms in which the two interfering states have opposite symmetry (one natural parity and the other unnatural parity) should play a major role. This suggests that three-body forces may well have a sizable effect on the axial polarization observables, arising through the interference between relatively large natural parity states and the smaller unnatural parity states.

To illustrate, we present the explicit results for the spin-$\frac{1}{2}$ analyzing powers. Working from Eq. (12), we obtain

$$\sigma T_{kq} = \sum_{\beta\beta' L_x L_y \mathcal{L}} C_{L_x L_y \mathcal{L}}^{\beta\beta'} B_{L_x L_y \mathcal{L}}^{q}(\hat{\mathbf{k}}_x, \hat{\mathbf{k}}_y), \qquad (13)$$

where B is defined in Eq. (4) and where we have used the symbol β to represent collectively the quantum numbers l, s, J, and α. The expansion coefficients C in Eq. (13) are given by

$$C^{\beta\beta'}_{L_xL_y\mathcal{L}} \propto \sum_I \delta_{S,S'}\delta_{S_x,S_x'}U^J_{\alpha,sl}U^{J'*}_{\alpha',s'l'}\langle l0,l'0|I0\rangle\langle l_x0,l_x'0|L_x0\rangle\langle l_y0,l_y'0|L_y0\rangle\langle\mathcal{L}-q,kq|I0\rangle \times W(LJL'J';S\mathcal{L})W(s\tfrac{1}{2}s'\tfrac{1}{2};1k)\begin{Bmatrix} l & s & J \\ l' & s' & J' \\ I & k & \mathcal{L}\end{Bmatrix}\begin{Bmatrix} l_x & l_y & L \\ l_x' & l_y' & L' \\ L_x & L_y & \mathcal{L}\end{Bmatrix}. \quad (14)$$

There are many restrictions on the various quantum numbers that appear in Eq. (14). From the Clebsch-Gordan coefficients we find that the combinations $l + l' + I$, $l_x + l_x' + L_x$, and $l_y + l_y' + L_y$ must all be even. In addition, parity conservation requires that U is zero unless the initial and final states have equal parity, leading to the requirement that $l + l_x + l_y$ and $l' + l_x' + l_y'$ must be even.

If we now turn to the special case of the longitudinal analyzing power ($k = 1$, $q = 0$), we have in addition that $\mathcal{L} + k + I$ must be even. Combining these results we conclude that for A_z the coefficients C are zero unless $L_x + L_y + \mathcal{L}$ is odd.

We may now use this result to understand which states α and α' may contribute to the coefficient of a given angular function $B_{L_xL_y,\mathcal{L}}$ in Eq. (13). The relevant angular momentum coupling is shown in Eq. (14). By making use of the fact that in the 9-j symbols each row and column must satisfy the usual triangle relation, we see that for a particular value of $\mathcal{L}$ a pair of states α and α' (which are characterized, respectively, by orbital quantum numbers l_x,l_y,L and l_x',l_y',L') may contribute only if L and L' satisfy a triangle relation with $\mathcal{L}$. Thus, for a given angular function $B_{L_xL_y,\mathcal{L}}$, the two interfering states must have $L + L'$ of at least $\mathcal{L}$. However, we also know that $l_x + l_x' + L_x$ and $l_y + l_y' + L_y$ are both even and that $L_x + L_y + \mathcal{L}$ is odd. From this it follows that for the lowest possible choice of the combined angular momentum of the two states (i.e., for $L + L' = \mathcal{L}$), one of the two states must have natural parity and the other unnatural parity. (The general result is that whenever $L + L' + \mathcal{L}$ is even the two states must have opposite symmetry.)

For example, an angular function with $\mathcal{L} = 2$ can arise from S-D or P-P interference if one of the two states has unnatural parity, but requires P-D interference if both have natural parity. This suggests that the contributions to A_z involving cross terms between natural and unnatural parity states may be important, particularly for low energies where one expects the reaction to be dominated by low angular momentum values.

In conclusion, we believe there are good reasons to expect that the quantities referred to here as the axial polarization observables will have an enhanced sensitivity to the presence of nuclear three-body forces. Clearly, what is needed at this point is to test this conjecture by carrying out full three-body calculations with and without $3N$ forces. If the detailed calculations indicate a high or even moderate degree of sensitivity, measurements of the axial polarization observables could well turn out to play a key role in determining the nature of the nuclear three-body force. We should also keep in mind that breakup reactions offer a wealth of kinematic conditions to be explored, and that there are many axial polarization observables. One can hope that detailed calculations will show us how to choose observables and kinematics to maximize the sensitivity to the $3N$ potential.

To our knowledge, there are no existing measurements of axial polarization observables for N-d breakup at any energy. Such measurements would be relatively straightforward, and, in fact, plans for an experiment to measure the longitudinal analyzing power in p-d breakup are currently in progress at Wisconsin.

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