HOMEWORK 10 SOLUTIONS

9-23) (a) From the periodic table the masses are 39.1 u for K and 35.45 u for Cl

$$\mu = \frac{m_1 m_2}{m_1+m_2} = \frac{(35.45)(39.1)}{(35.45)+(39.1)} = \boxed{18.59\,u}$$

(b) $I = \mu r_0^2$ and we have

$$\frac{\hbar^2}{2I} = \frac{\hbar^2}{2\mu r_0^2} = 1.43\times10^{-5}\,eV = E_{or}$$

$$\Rightarrow \quad r_0^2 = \frac{\hbar^2}{2\mu E_{or}} = \frac{(\hbar c)^2}{2E_{or}\mu c^2}$$

Now

$$1u\cdot c^2 = 931.5\,MeV \quad \Rightarrow \quad r_0^2 = \frac{(197.33\,eV\cdot nm)^2}{2(1.43\times10^{-5}eV)(18.59)(9.315\times10^{8}eV)} = 7.86\times10^{-2}\,nm^2$$

$$\boxed{r_0 = 0.280\,nm}$$

9-24) (a) For $H^{35}Cl$ $\quad f = 8.97\times10^{13}/s = \frac{1}{2\pi}\sqrt{\frac{k}{\mu}} \quad \Rightarrow \quad k = (2\pi f)^2\cdot\mu$

$$\mu = \frac{m_1 m_2}{m_1+m_2} = \frac{(1.008\,u)(35\,u)}{(1.008\,u)+(35\,u)} = 0.980\,u = (0.980)(1.66\times10^{-27}kg)$$

$$K = (2\pi\cdot 8.97\times10^{13}/s)^2(0.980)(1.66\times10^{-27}kg) = \boxed{516.6\,kg/s^2} \text{ (or N/m)}$$

(b) For $K^{79}Br$

$$\mu = \frac{(39.1)(79)}{(39.1)+(79)} = 26.15\,u$$

$$K = (2\pi\cdot 6.93\times10^{12}/s)^2(26.15)(1.66\times10^{-27}kg) = \boxed{82.3\,kg/s^2}$$

9-29) We have levels at $E = 0, 3.8eV, 4.2eV, 7.2eV, 7.6eV$.

(a) For $\lambda = 3100nm$ $\quad E_\gamma = hc/\lambda = 1240eV\cdot nm/3100nm = 0.4eV$, which is the energy difference between states 2 and 3 and also between states 4 and 5

i) For absorption we can have $\boxed{2 \to 3 \text{ and } 4 \to 5}$

ii) For stimulated emission $\boxed{3 \to 2 \text{ and } 5 \to 4}$

iii) Any downward transition is possible:

$$\boxed{2\to1,\ 3\to2,\ 3\to1,\ 4\to3,\ 4\to2,\ 4\to1,\ 5\to4,\ 5\to3,\ 5\to2,\ 5\to1}$$

(b) For $T=0$ only the ground state is populated $\Rightarrow$ none of the transitions occur.

(c) For $\lambda = 326.3\text{ nm}$ $\quad E = \dfrac{1240\text{ eV.nm}}{326.3\text{ nm}} = 3.80\text{ eV}$

i) Here we can have $\boxed{1 \to 2 \text{ and } 2 \to 5}$

ii) " $\boxed{2 \to 1 \text{ and } 5 \to 2}$

iii) The answer is the same as in part (a). All downward transitions are possible

Now for $T=0$ only state 1 is populated, so we get $\boxed{1\to2}$ by absorption as the only possible transition.

(d) For

$4\to3$	$E = 3.0\text{ eV}$	$\lambda = 413.3\text{ nm}$
$4\to2$	$E = 3.4\text{ eV}$	364.7 nm
$4\to1$	$E = 7.2\text{ eV}$	172.2 nm

9-32) (a) The rotational energy parameter is $E_{or} = \dfrac{\hbar^2}{2I}$. For equal masses $\mu = m_1 m_2/(m_1+m_2) = m/2$ (m is the mass of 1 atom)

$$I = \mu r_0^2 = \tfrac{1}{2} m r_0^2$$

$$E_{or} = \frac{\hbar^2}{2(\frac{1}{2}mr_0^2)} = \frac{\hbar^2c^2}{(mc^2)r_0^2} = \frac{(197.3\text{ eV.nm})^2}{(19.0)(9.315\times10^{8}\text{ eV})(0.14\text{ nm})^2} = \boxed{1.123\times10^{-4}\text{ eV}}$$

$\ell=3$ —— $1.35\times10^{-3}\text{ eV}$

$\ell=2$ —— $6.74\times10^{-4}\text{ eV}$

$\ell=1$ —— $2.25\times10^{-4}\text{ eV}$

$\ell=0$ —— 0

In general $E = \ell(\ell+1)E_{or}$

$= 0,\ 2E_{or},\ 6E_{or},\ 12E_{or}$

(b) With the selection rule $\Delta \ell = \pm 1$ we can have

$\ell = 3 \rightarrow \ell = 2$ $\quad E_\gamma = 6E_{or} = 6.74 \times 10^{-4}\,eV$

$\ell = 2 \rightarrow \ell = 1$ $\quad E_\gamma = 4E_{or} = 4.49 \times 10^{-4}\,eV$

$\ell = 1 \rightarrow \ell = 0$ $\quad E_\gamma = 2E_{or} = 2.25 \times 10^{-4}\,eV$

$$\boxed{\begin{aligned}\lambda &= 1.84\ mm\\ \lambda &= 2.76\ mm\\ \lambda &= 5.52\ mm\end{aligned}}$$

9-36) (a) If $\lambda = 420$ nm then the energy difference between the states is $E = hc/\lambda = (1240\,eV \cdot nm)/420\,nm = 2.95\,eV$

For $T = 297K$ $\quad kT = (8.62 \times 10^{-5}\,eV/K)(297K) = 0.0256\,eV.$

$$\frac{N_2}{N_1} = e^{-E/kT} \qquad N_2 = (2.5 \times 10^{21})\, e^{-2.95/0.0256}$$

$$= (\ \text{"}\ )\, 8.8 \times 10^{-51} \Rightarrow N_2 \simeq 0$$

(b) If we now pump 1.8×10^{21} atoms to the upper state we get a population inversion with

$$N_2 = 1.8 \times 10^{21}$$
$$N_1 = 0.7 \times 10^{21}$$

When the laser fires we get both stimulated emission + absorption (at equal probabilities) which leads to $N_1 = N_2 = 1.25 \times 10^{21}$ ⇒ we get

$$1.8 \times 10^{21} - 1.25 \times 10^{21} = 0.55 \times 10^{21} \text{ decays.}$$

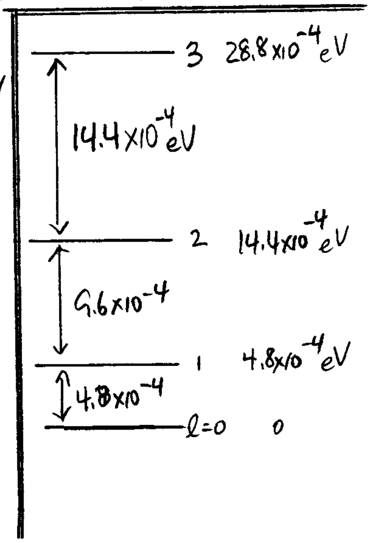
$$\Rightarrow E = (0.55 \times 10^{21})(2.95\,eV) = 1.62 \times 10^{21}\,eV = \boxed{260\text{ Joules}}$$

9-42) (a) $\lambda = 0.86\,mm \Rightarrow E_\gamma = \dfrac{hc}{\lambda} = \dfrac{1240\,eV \cdot nm}{0.86 \times 10^6\,nm} = 1.44 \times 10^{-3}\,eV$

$\lambda = 1.29\,mm \Rightarrow E = \dfrac{1240\,eV \cdot nm}{1.29 \times 10^6\,nm} = 9.61 \times 10^{-4}\,eV$

$\lambda = 2.59\,mm \Rightarrow E = 4.79 \times 10^{-4}\,eV$

The energies go like $E_0, 2E_0, 3E_0$ etc which means rotational motion.



(b) So the lowest energy is $2E_{0r} = 2\left(\frac{\hbar^2}{2I}\right) = \frac{\hbar^2}{I} = 4.8\times10^{-4}\,eV$

$$I = \mu r_0^2 = \frac{\hbar^2}{4.8\times10^{-4}\,eV} \qquad \mu = \frac{(12u)(16u)}{12u+16u} = 6.86\,u$$

$$r_0 = \left[\frac{(\hbar c)^2}{(6.86)(9.315\times10^{6}\,eV)(4.8\times10^{-4}\,eV)}\right]^{\frac{1}{2}} \qquad \boxed{r_0 = 0.113\,nm}$$

9-45) For HCl the reduced mass is $\mu = \frac{(1.008)(35.45)}{1.008+35.45} = 0.98\,u$

From Fig. 9-29, the frequency corresponding to $\hbar\omega$ appears to be about 8.65×10^{13} Hz. We have

$$f = \frac{1}{2\pi}\sqrt{k/\mu} \quad \Rightarrow \quad k = (2\pi\cdot f)^2\mu$$

$$k = (2\pi)^2(8.65\times10^{13}/s)^2(0.98)(1.66\times10^{-27}\,kg) \qquad \boxed{k = 480\ N/m}$$

I have no idea why the frequency from Fig 9-29 doesn't agree with the value in Table 9-7.