HOMEWORK 11 SOLUTIONS

10-3) We assume a potential of the form $U(r) = -\alpha \frac{e^2}{4\pi\epsilon_0} \cdot \frac{1}{r_0}\left[\frac{r_0}{r} - \frac{1}{n}\left(\frac{r_0}{r}\right)^n\right]$

For LiCl the constant α has the value 1.7476 and the equilibrium separation is $r_0 = 0.257$ nm. We want to choose n to get the correct value of U at $r = r_0$. The dissociation energy is $741\text{ kJ/mole} = 7.41\times10^5\text{ J/mole} \Rightarrow$

$$U(r_0) = \left[(7.41\times10^5\text{ J})/(6.02\times10^{23})\right] = 1.23\times10^{-18}\text{ J/pair}$$
$$= 7.68\text{ eV/ion pair.}$$

So we need

$$\alpha\left(\frac{e^2}{4\pi\epsilon_0}\right)\left(\frac{1}{r_0}\right)\left[1-\frac{1}{n}\right] = 7.68\text{eV}.$$

$$\left[1-\frac{1}{n}\right] = (7.68\text{eV})(0.257\text{nm})/(1.7476)(1.44\text{eV}\cdot\text{nm})$$
$$= 0.785$$
$$\Rightarrow \boxed{n = 4.6}$$

10-5) The dissociation energy of CsCl is 648 kJ/mole which translates to

$$(6.48\times10^5\text{ J/mole})/(6.02\times10^{23})/(1.602\times10^{-19}\text{ J/eV})$$
$$= 6.73\text{ eV/ion pair.}$$

According to the text, the dissociation energy is the energy to separate into ion pairs, and the cohesive energy is the energy needed to produce separated neutral atoms. From Table 9.1, Cs has an ionization energy of 3.89 eV and Cl has an electron affinity of 3.62 eV, so the net cohesive energy is

$$6.73\text{eV} + 3.62\text{eV} - 3.89\text{eV} = \boxed{6.46\text{eV /ion pair.}}$$

10-8) (a) The wire will have a cross sectional area of

$$A = \pi r^2 = \frac{\pi}{4}d^2 = \left(\frac{\pi}{4}\right)(0.163\text{cm})^2 = 0.021\text{ cm}^2$$

So $J = I/A = 1\,\text{mA} / 0.021\,\text{cm}^2 = \boxed{48\,\text{mA/cm}^2}$

(b) The density of free electrons in copper is the same as the density of atoms (1 free electron / atom) $\Rightarrow$ (see p. 447)

$$n = 8.47\times10^{22}/\text{cm}^3$$

So

$$v_d = J/e\cdot n = (48\times10^{-3}\,\text{A/cm}^2)/(8.47\times10^{22}/\text{cm}^3)(1.602\times10^{-19}\,\text{C})$$

$$\boxed{v_d = 3.5\times10^{-6}\,\text{cm/s}}$$

10-10) (a) For Ag $\rho = 10.5\,\text{g/cm}^3$ $M = 107.87\,\text{g/mole}$

$$\rho = [(10.5\,\text{g/cm}^3)/(107.87\,\text{g/mole})]\times(6.02\times10^{23}\,\text{atoms/mole})$$

$$\boxed{\rho = 5.86\times10^{22}/\text{cm}^3}$$

(b) For Au $\rho = 19.3\,\text{g/cm}^3$ $M = 196.97\,\text{g/mole}$

gives

$$\boxed{\rho = 5.90\times10^{22}/\text{cm}^3}$$

10-12) (a) For $\lambda = 0.37\,\text{nm}$ and $\langle v\rangle = 1.08\times10^5\,\text{m/s}$ we obtain

$$\sigma = \left(\frac{e^2}{m_e}\right) n\, \lambda/\langle v\rangle$$

$$= (1.6\times10^{-19}\,\text{C})^2(8.47\times10^{22}/\text{cm}^3)(0.37\,\text{nm})/(9.11\times10^{-31}\,\text{kg})(1.08\times10^5\,\tfrac{\text{m}}{\text{s}})$$

$$= (1.6\times10^{-19}\,\text{C})^2(8.47\times10^{28}/\text{m}^3)(3.7\times10^{-10}\,\text{m})/(9.11\times10^{-31}\,\text{kg})(1.08\times10^5\,\tfrac{\text{m}}{\text{s}})$$

$$\boxed{\sigma = 8.17\times10^6/\Omega\cdot\text{m} \Rightarrow \rho = 1/\sigma = 1.22\times10^{-7}\,\Omega\cdot\text{m}}$$

(b) In the calculation above we assume $\langle v\rangle = \left[\frac{8}{\pi}\frac{kT}{m}\right]^{\frac{1}{2}} \Rightarrow \langle v\rangle \propto \sqrt{T}$

Everything else stays the same $\Rightarrow$ $\rho \propto \sqrt{T}$ and $\sigma \propto 1/\sqrt{T}$

$\Rightarrow$

$$\sigma_{200} = \sqrt{\tfrac{300}{200}}\,\sigma_{300} = \sqrt{\tfrac{3}{2}}\,(8.17\times10^6/\Omega\cdot\text{m}) = \boxed{1.00\times10^7/\Omega\cdot\text{m} = \sigma} \quad 200\text{K}$$

$$\boxed{1.00\times10^{-7}\,\Omega\cdot\text{m} = \rho}$$

(c) $\sigma_{100} = \sqrt{2}\,\sigma_{200}$ $\boxed{= 1.41\times10^7/\Omega\cdot\text{m}}$ 100K

$$\boxed{\rho = 0.71\times10^{-7}\,\Omega\cdot\text{m}}$$

10-15) From 10-10 the density of electrons is $5.90\times10^{22}/cm^3$.

(a) From 10-35 the Fermi energy is

$$E_F = \frac{h^2}{2m}\left(\frac{3}{8\pi}\cdot\frac{N}{V}\right)^{2/3}$$

$$\frac{N}{V} = (5.9\times10^{22}/cm^3)\cdot\left(\frac{1cm}{10^7 nm}\right)^3 = 59/nm^3$$

$$E_F = \frac{(1240 eV\cdot nm)^2}{2(5.11\times10^5 eV)}\left(\frac{3}{8\pi}\cdot 59/nm^3\right)^{2/3} \qquad \boxed{E_F = 5.53\, eV}$$

(b) The corresponding temperature is defined by $kT_F = E_F$

$$T_F = E_F/k = (5.53\, eV)/(8.617\times10^{-5} eV/K) = \boxed{6.4\times10^4 K}$$

10-17) We calculate the "Fermi speed" using $\frac{1}{2}m_e v_F^2 = E_F \Rightarrow v_F = [2E_F/m_e]^{\frac{1}{2}}$

$\Rightarrow \quad v_F = c\,[2E_F/m_ec^2]^{\frac{1}{2}}$

(a) For Na $\quad E_F = 3.26\, eV \qquad v_F = (3\times10^8 m/s)\,[2(3.26eV)/5.11\times10^5 eV]^{\frac{1}{2}}$

$$\boxed{v_F = 1.07\times10^6\ m/s}$$

(b) For Au $\quad E_F = 5.55eV \Rightarrow \boxed{v_F = 1.40\times10^6\ m/s}$

(c) For Sn $\quad E_F = 10.3eV \Rightarrow \boxed{v_F = 1.90\times10^6\ m/s.}$

10-18) In the quantum theory $\sigma = \frac{e^2}{m_e} n\, \lambda/v_F$ so with $\sigma = \frac{1}{\rho}$ we have

$$\lambda = m_e\sigma v_F/ne^2 = \left(\frac{m_e v_F}{ne^2}\right)\frac{1}{\rho}$$

We can get the charge carrier densities from table 10-3.

(a) Na:

$$\lambda = (9.11\times10^{-31}kg)(1.07\times10^6 m/s)/(1.602\times10^{-19}C)^2(2.65\times10^{28}/m^3)(4.2\times10^{-8}\Omega\cdot m)$$

$$= 3.42\times10^{-8} m = \boxed{34.2\, nm}$$

(b) Au: using $v = 1.4\times10^6 m/s$ and $\rho = 2.04\times10^{-8}\Omega\cdot m \Rightarrow \boxed{\lambda = 41.4\, nm}$

(c) Sn: " $1.9\times10^6 m/s$ " $\rho = 10.6\times10^{-8}\Omega\cdot m \Rightarrow \boxed{\lambda = 4.3\, nm}$

10-20) We have total energy $U = \frac{3}{5} N E_F + \left(\frac{\pi^2}{4}\right) N \frac{kT}{E_F} \cdot kT$, so the average energy is

$$\langle E \rangle = \frac{3}{5} E_F + \frac{\pi^2}{4} \frac{(kT)^2}{E_F}.$$

For copper $E_F = 7.06\,eV$, $kT = (8.617 \times 10^{-5} eV/K) \cdot (300K)$
$= .026\,eV$

$$\Rightarrow \quad \langle E \rangle = (0.6)(7.06)eV + \left(\frac{\pi^2}{4}\right) \frac{(.026eV)^2}{7.06eV} = 4.236eV + .00024eV$$

$$\boxed{\langle E \rangle = 4.24eV}$$

This is basically the same as $\langle E \rangle$ at $T=0$ and large compared to the classical value $\langle E \rangle = 3/2\, kT = 0.039eV$.

10-38) According to the formula we found in class, the fraction of the electrons above E_F is about $\frac{3}{2} \ln 2 \frac{kT}{E_F} \simeq \frac{kT}{E_F}$.

For copper $E_F \simeq 7eV \Rightarrow$

(a) $T = 300\,K$ $\quad \boxed{F = .026eV/7eV = 0.0038}$

(b) $T = 1000\,K$ $\quad \boxed{F = .086eV/7eV = 0.0128}$

10-39) (a) We have alternating + and − charges separated by r_0. A given + ion has two − neighbors at r_0, two + neighbors at $2r_0$, etc. So

+ − + − ⊕ − + − ...

$$V = -\frac{e^2}{4\pi\epsilon_0} \cdot \frac{1}{r_0}(2) + \frac{e^2}{4\pi\epsilon_0} \cdot \left(\frac{1}{2r_0}\right) \cdot 2 - \frac{e^2}{4\pi\epsilon_0} \cdot \left(\frac{1}{3r_0}\right) \cdot 2 + \cdots$$

$$\boxed{V = -2 \frac{e^2}{4\pi\epsilon_0} \cdot \frac{1}{r_0}\left(1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \cdots\right)}$$

(b) Given that $\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \cdots$ we have

$$\ln(1+1) = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \cdots$$

So

$$V = -2 \frac{e^2}{4\pi\epsilon_0} \cdot \frac{1}{r_0} (\ln 2) = -2\ln 2 \frac{e^2}{4\pi\epsilon_0} \frac{1}{r_0} = -\alpha \frac{e^2}{4\pi\epsilon_0} \cdot \frac{1}{r_0}$$

where

$$\alpha = 2 \ln 2$$