# HOMEWORK 12 SOLUTIONS

**10-25)** (a) Each side of the crystal is $100\,nm = 100\times10^{-9}\,m = 10^{-7}\,m = 10^{-5}\,cm$,
So $V = d^3 = 10^{-15}\,cm^3$. We have $2.33\,g/cm^3$ with atomic mass $28.09\,g/mole$

$$N = (10^{-15}\,cm^3)\cdot(2.33\,g/cm^3)\cdot(6.02\times10^{23}\,atoms/mole)/(28.09\,g/mole)$$

$$N = 5.00\times10^{7}$$

(b) We have $4N = 2\times10^{8}$ states total in $13\,eV$, so the density of states is

$$2\times10^{8}/13 = 1.54\times10^{7}/eV$$

$$AVE\ \Delta E = 1/1.54\times10^{7} \qquad \boxed{\Delta E = 6.5\times10^{-8}\,eV}$$

**10-28)** Aluminum is one below Si in the periodic table so it has valence 3
$\Rightarrow$ excess holes $\Rightarrow$ $\boxed{\text{p-type}}$
Phosphorus is one above Si $\Rightarrow$ extra electrons $\Rightarrow$ $\boxed{\text{n-type.}}$

**10-31)** $I = I_0\left(e^{eV_B/kT} - 1\right)$

(a) $eV_B/kT = 10 \Rightarrow eV_B = 10\,kT$.
At $300\,K$ $\quad kT = (8.617\times10^{-5}\,eV/K)\cdot(300\,K) = 0.026\,eV.$

$\Rightarrow$

$$eV_B = 0.26\,eV \qquad \boxed{V_B = 0.26\text{ volts}}$$

(b) $eV_B/kT = 0.1 \Rightarrow \boxed{V_B = 0.0026\text{ volts}}$

**10-33)** $I(0.1) = I_0\left(e^{0.1\,eV/.026\,eV} - 1\right) = 45.8\,I_0$

$I(0.2) = I_0\left(e^{0.2\,eV/.026\,eV} - 1\right) = 2190\,I_0$

$$\boxed{\frac{I(0.2)}{I(0.1)} = 47.8}$$

So the current increases by a factor of 48.

10-34) For lead $\alpha = 0.49$ (Table 10-7) and $T_c = 7.196\,K$ for ordinary lead ($M = 207.19$ from the periodic table). The corresponding masses for $^{206}Pb$, $^{207}Pb$ and $^{208}Pb$ are 205.974, 206.976 and 207.977. The isotope effect equation is $M^{\alpha} T_c = \text{constant} = (207.19)^{\alpha} \cdot (7.196K)$

$$^{206}Pb \qquad T_c = \left(\frac{207.19}{205.974}\right)^{0.49} \cdot 7.196K = \boxed{7.217K}$$

$$^{207}Pb \qquad T_c = \left(\frac{207.19}{206.976}\right)^{0.49} \cdot \quad '' \quad = \boxed{7.200K}$$

$$^{208}Pb \qquad T_c = \left(\frac{207.19}{207.977}\right)^{0.49} \cdot \quad '' \quad = \boxed{7.183K}$$

10-36) (a) $E_g = 3.5\,kT_c$. From Table 10-6 $T_c = 3.722$ so we predict

$$E_g = (3.5)(8.617\times10^{-5}\,eV/K)(3.722K) = \boxed{1.12\times10^{-3}\,eV}$$

This is about a factor of 2 larger than the actual measured value.

(b) For $E = 6\times10^{-4}\,eV$ $\quad \frac{hc}{\lambda} = E$ gives $\lambda = \frac{hc}{E}$

$$\lambda = \left(\frac{1240\,eV\cdot nm}{6\times10^{-4}\,eV}\right) = 2.07\times10^{6}\,nm$$

$$\boxed{\lambda = 2.07\ mm}$$

10-45) The idea is that the electron moves in an orbit that is similar to the first Bohr orbit in hydrogen. In the normal hydrogen calculation

$$a_0 = \frac{\epsilon_0 h^2}{\pi m_e e^2} = \frac{4\pi\epsilon_0 \hbar^2}{e^2 m_e} = \left(\frac{4\pi\epsilon_0}{e^2}\right)\frac{(\hbar c)^2}{m_e c^2}$$

$$= \left(\frac{1}{1.44\,eV\cdot nm}\right)\frac{(197.3\,eV\cdot nm)^2}{5.11\times10^{5}\,eV} = 0.0529\,eV.$$

In the solid $\epsilon_0 \to K\epsilon_0$ and we use the "effective mass" of the electron $\Rightarrow$

Si $\quad r = (K/0.2)\, a_0 = \left(\frac{12}{0.2}\right) a_0 = \boxed{3.17\ nm}$

Ge $\quad r = (K/0.1)\, a_0 = \left(\frac{16}{0.1}\right) a_0 = \boxed{8.46\,nm}$

10-46) (a) The electrical resistivity is given in 10-40 as $\rho = \frac{m_e v_F}{ne^2} \cdot \frac{1}{\lambda}$

In copper $v_F = [2E_F/m_e]^{\frac{1}{2}} = [2E_F/m_e c^2]^{\frac{1}{2}} \cdot c$

$= (2(7eV))/5.11\times10^5 eV)^{1/2} \cdot 3\times10^8\ m/s = 1.57\times10^6\ m/s.$

Now

$$\rho = \frac{m_e v_F}{ne^2}\left(\frac{1}{\lambda_m} + \frac{1}{\lambda_i}\right)$$

So

$$(1.2\times10^{-8}\ \Omega\cdot m) = \frac{m_e v_F}{ne^2} \cdot \frac{1}{\lambda_i}$$

Using $n = 8.47\times10^{22}\ e/cm^3$ in copper we get

$$\lambda_i = \frac{(9.11\times10^{-31}kg)(1.57\times10^6 m/s)}{(8.47\times10^{28}/m^3)(1.6\times10^{-19}C)^2(1.2\times10^{-8}\Omega\cdot m)} = 5.5\times10^{-8}\ m = \boxed{55\,nm.}$$

(b) From 10-19 $\quad \lambda = 1/n_a\pi r^2 = 1/n_a \frac{\pi}{4} d^2$ where $n_a$ is now the density of impurities $= 0.01 \times 8.47\times10^{22}/cm^3 = 8.47\times10^{20}/cm^3$

$= 0.847/nm^3$

$$\left(\frac{\pi}{4}d^2\right) = \frac{1}{\lambda n_a} = \frac{1}{(55nm)(0.847/nm^3)} = 0.021\ nm^2$$

$$d^2 = 0.027\,nm^2 \Rightarrow \boxed{d = 0.165\ nm} \Rightarrow \text{reasonable}$$