HOMEWORK 3 SOLUTIONS

E4) (a) The mean free path is $l = 1/n\sigma$ where n is the number of atoms per unit volume and σ is the cross section. We will use

$$\sigma = \pi d^2 = 4\pi r^2$$

where $r = 0.19\,\text{nm}$. We get n from the ideal gas law

$$PV = nRT$$

where n here is the number of moles. If N is the number of atoms then

$$N = nN_A \Rightarrow PV = \left(\frac{N}{N_A}\right)RT$$

$$n = \text{density} = \frac{N}{V} = \frac{P \cdot N_A}{RT}$$

Thus

$$l = \frac{1}{n\sigma} = \frac{RT}{P \cdot N_A} \cdot \frac{1}{4\pi r^2} = \frac{(8.314\ \text{J/mole}\cdot\text{K})(273\text{K})}{(1.01\times10^5\ \text{N/m}^2)(6.02\times10^{23}/\text{mole})} \frac{1}{4\pi(1.9\times10^{-10}\text{m})^2}$$

$$\boxed{l = 8.23\times10^{-8}\ \text{m}} \sim 10^{-7}\ \text{m}$$

The radius is $1.9\times10^{-10}\,\text{m}$ so l is about 500 times the radius

The volume per atom is

$$V/N = RT/P\cdot N_A \Rightarrow L = \left[\frac{RT}{P \cdot N_A}\right]^{\frac{1}{3}} = 0.72\times10^{-9}\ \text{m}$$

So l is about 10 times the average distance between atoms

(b) From our class notes

$$X_{RMS} = \left[\tfrac{2}{3}\, l\, \bar{v}\, t\right]^{\frac{1}{2}}$$

Here

$$\bar{v} = \left[\frac{8}{\pi}\frac{kT}{m}\right]^{\frac{1}{2}} = \left[\frac{8}{\pi}\frac{(kN_A)\cdot T}{m \cdot N_A}\right]^{\frac{1}{2}} = \left[\frac{8}{\pi}\frac{(8.314\,\text{J/K})(273\text{K})}{(39.95\times10^{-3}\,\text{kg})}\right]^{\frac{1}{2}}$$

$$= 380\ \text{m/s}$$

So

$$X_{RMS} = \left[\tfrac{2}{3}(8.23\times10^{-8}\text{m})(380\ \text{m/s})(10\text{s})\right]^{\frac{1}{2}} = 0.0144\ \text{m} = \boxed{1.44\,\text{cm}}$$

The distance traveled is $\bar{v}t = \boxed{3800\,\text{m}}$

E5) (a) The thermal conductivity is given by $K = \frac{1}{3} l n \bar{v} \left({C_v}/{N_A} \right)$

From E4, $l = 8.23 \times 10^{-8}$ m and $\bar{v} = 380$ m/s. According to the equipartition theorem $C_v = \frac{3}{2} R$ (argon is monatomic). Finally

$$n = \frac{N}{V} = \frac{P N_A}{RT}$$

$\Rightarrow$

$$K = \frac{1}{3} l \left(\frac{P \cancel{N_A}}{\cancel{R} T} \right) \bar{v} \left(\frac{\frac{3}{2} \cancel{R}}{\cancel{N_A}} \right)$$

$$= \frac{1}{2} (8.23 \times 10^{-8}\,\text{m})(1.01 \times 10^{5}\,\text{N/m}^2)(380\,\text{m/s}) / (273\,\text{K})$$

$$\boxed{K = 5.78 \times 10^{-3}\ \text{N/s}\cdot\text{K} = 5.78 \times 10^{-3}\ \frac{\text{W}}{\text{m}\cdot\text{K}}}$$

(b) If we double the density n increases by a factor of 2 but l decreases by a factor of 2. If T is constant $\bar{v}$ doesn't change $\boxed{K \text{ is unchanged.}}$

(c) If n is fixed and T doubles the only change is that $\bar{v}$ increases. Now $\bar{v} \propto \sqrt{T}$ so

$\boxed{K \text{ increases by factor } \sqrt{2}}$

3-18) According to Eq. 3-32 the average energy is $\bar{E} = \dfrac{hc/\lambda}{e^{hc/\lambda kT} - 1}$ which we can write as

$$\bar{E} = kT \frac{hc/\lambda kT}{e^{hc/\lambda kT} - 1}$$

$\lambda = 10\, hc/kT \Rightarrow \dfrac{hc}{\lambda kT} = \dfrac{1}{10}$

$$\bar{E} = kT \frac{(0.1)}{e^{0.1} - 1} \Rightarrow \boxed{\bar{E} = 0.951\, kT}$$

$\bar{E} \simeq kT$

$\lambda = 0.1\, hc/kT \Rightarrow \dfrac{hc}{\lambda kT} = 10$

$$\bar{E} = kT \frac{10}{e^{10} - 1} \Rightarrow \boxed{\bar{E} = 4.54 \times 10^{-4}\, kT}$$

$\bar{E} \ll kT$

3-21) (a) From 3-20 $\lambda_m T = 2.898\times10^{-3}\,m\cdot K$. So for $T = 3300K$

$$\lambda_m = \frac{.002898}{3300}\,m = 8.78\times10^{-7}\,m \qquad \boxed{\lambda_m = 878\,nm}$$

The corresponding frequency is

$$f_m = c/\lambda_m = 3\times10^{8}\,m/s \,/\, 8.78\times10^{-7}\,m \Rightarrow \boxed{f_m = 3.42\times10^{14}/s}$$

(b) Each photon has energy $E = hf = hc/\lambda$ so

$$E = (1240\,eV\cdot nm)/878\,nm = 1.41\,eV$$
$$= 2.262\times10^{-19}J$$

If we assume all of the 40 W goes into light (probably not really true) then we have 40 J/s ⇒

$$n = (40J/s)/(2.62\times10^{-19}J/\text{photon}) = \boxed{1.77\times10^{20}/s}$$

(c) Assume that the photons are emitted uniformly in all directions. At a distance of 5 m the photons illuminate the surface of a sphere which has area $4\pi R^2 = 4\pi(5m)^2 = 314\,m^2$. Thus we have

$$\frac{1.77\times10^{20}}{314}\ \text{photons}/m^2\cdot s$$

If your pupil diameter is 5.0 mm, it's area is $\pi r^2 = \frac{\pi}{4}d^2$ and the number of photons per second that hit that area is

$$\left(\frac{1.77\times10^{20}/s}{4\cancel{\pi}R^2}\right)\cdot\left(\frac{\cancel{\pi}}{4}d^2\right) = (1.77\times10^{20}/s)\left(\frac{.005\,m}{4\cdot5m}\right)^2 = \boxed{1.1\times10^{13}/s}$$

3-23) We need the photons to have at least 0.68eV. The photon energy is $E = hf = hc/\lambda$

$$\Rightarrow \quad \lambda = \frac{hc}{E} = \frac{1240\,eV\cdot nm}{0.68\,eV} \Rightarrow \boxed{\lambda = 1824\,nm}$$

The visible region is 400-700 nm ⇒ this is beyond the red end ⇒ I would say <u>infrared</u>.

3-27) (a) For electrons to escape the surface we need $hf > \phi$.

$\Rightarrow$ the threshold frequency is $hf = \phi$

$$\Rightarrow \quad f = \frac{\phi}{h} = \frac{4.22\,eV}{4.136 \times 10^{-15}\,eV\cdot s} \Rightarrow \boxed{f = 1.02 \times 10^{15}\,/s}$$

(b) For light of wavelength $\lambda = 560$ nm we can calculate the photon energy as follows

$$E = hf = \frac{hc}{\lambda} = \frac{1240\,eV\cdot nm}{560\,nm} = \underline{2.21\,eV}$$

Since

$\underline{E < \phi \text{ photoelectrons are not emitted.}}$

3-36) Postponed.