HOMEWORK 5 SOLUTIONS

5-3 If the wavelength is 0.04 nm then the momentum is

$$pc = \frac{hc}{\lambda} = \frac{1240\,\mathrm{eV\cdot nm}}{.04\,\mathrm{nm}} = 31{,}000\,\mathrm{eV}$$

and

$$KE = \frac{p^2}{2m} = \frac{(pc)^2}{2mc^2} = \frac{(3.1\times10^4\,\mathrm{eV})^2}{2(5.11\times10^5\,\mathrm{eV})} = 940\,\mathrm{eV}$$

so

$$\boxed{V_0 = 940\,\mathrm{volts}}$$

5-5) If we take room temperature to be 293 K then $kT = .02525\,\mathrm{eV}$

$\Rightarrow \quad E = \frac{3}{2}kT = 0.0379\,\mathrm{eV} = 6.07\times10^{-21}\,\mathrm{J}.$

The mass of a nitrogen molecule is

$$m = 28\,\mathrm{g}/6.02\times10^{23} = 4.65\times10^{-23}\,\mathrm{g} = 4.65\times10^{-26}\,\mathrm{kg}$$

and so the momentum is

$$p = [2mE]^{\frac{1}{2}} = [2(4.65\times10^{-26}\,\mathrm{kg})(6.07\times10^{-21}\,\mathrm{J})]^{\frac{1}{2}} = 2.38\times10^{-23}\,\mathrm{kg\cdot m/s}$$

and so

$$\lambda = h/p = \frac{6.626\times10^{-34}\,\mathrm{J.s}}{2.38\times10^{-23}\,\mathrm{kg\cdot m/s}} = \boxed{2.78\times10^{-11}\,\mathrm{m}} = .00278\,\mathrm{nm}$$

5-11) First we need to find the wavelength. The neutron mass is

$$m = 939.57\,\mathrm{MeV}/c^2$$

so

$$pc = [2Emc^2]^{\frac{1}{2}} = [2(0.02\,\mathrm{eV})(9.3957\times10^8\,\mathrm{eV})]^{\frac{1}{2}} = 6130\,\mathrm{eV}$$

$\Rightarrow$

$$\lambda = \frac{h}{p} = \frac{1240\,\mathrm{eV\cdot nm}}{6130\,\mathrm{eV}} = 0.202\,\mathrm{nm}$$

Then we need $n\lambda = D\sin\phi$

$$\sin\phi = \frac{\lambda}{D} = \frac{0.202\,\mathrm{nm}}{0.215\,\mathrm{nm}} \Rightarrow \boxed{\phi = 70.2^\circ}$$

E-6) From our class notes, the coherent scattering condition is $n\lambda = 2d\sin\theta$ where $d = a\sin\alpha$. From the geometry $\alpha = \phi/2 = 26.5°$ and $\theta = \pi/2 - \alpha = \pi/2 - \phi/2$, and so we have

$$n\lambda = 2a\sin\phi/2\,\sin(\pi/2 - \phi/2)$$
$$= 2a\sin\phi/2\,\cos\phi/2$$

Assuming $n=1$ and $a = 0.2$ nm we have

$$\lambda = (2)(0.2\,\text{nm})\sin(26.5°)\cos(26.5°)$$
$$= 0.1597\,\text{nm}$$

The corresponding momentum is

$$pc = hc/\lambda = 1240\,\text{eV}\cdot\text{nm}/0.1597\,\text{nm} = 7763\,\text{eV}.$$

(a) $E = p^2/2m = (pc)^2/2mc^2$
$$= (7763\,\text{eV})^2/2(9.3957\times10^8\,\text{eV}) = \boxed{0.0321\,\text{eV}}$$

(b) $E = p^2/2m = (7763\,\text{eV})^2/2(5.11\times10^5\,\text{eV}) = \boxed{59.0\,\text{eV}}$

(c) $E = pc = \boxed{7763\,\text{eV}}$

5-17) (a) $y_1 + y_2 = 0.002\,[\cos(8.0x - 400t) + \cos(7.6x - 380t)]$

$$= \boxed{0.004\,[\cos(0.2x - 10t)\cdot\cos(7.8x - 390t)]}$$

(b) The phase velocity is

$$v_p = \bar{\omega}/\bar{k} = (390/s)/(7.8/m) = \boxed{50\,m/s}$$

(c) $$v_g = \frac{\Delta\omega}{\Delta k} = \frac{10/s}{0.2/s} = \boxed{50\,m/s}$$

(d) The envelope of the wave is the function $\cos(0.2x - 10t)$. The cos functions has zeros at $\pi/2, 3\pi/2, \cdots$ and so at $t=0$ the zeros occur at

$$0.2x = \pi/2,\quad 0.2x = 3\pi/2,\quad 0.2x = 5\pi/2 \text{ etc.}$$
$$x = \pi/.4,\quad 3\pi/.4,\quad 5\pi/.4 \text{ etc.}$$

The distance between zeros is $\boxed{\Delta x = 2\pi/0.4 = 15.7\,m}$

We had $\Delta k = 0.4/m$ so $\boxed{\Delta k \cdot \Delta x = 6.28 = 2\pi}$

E-7) We have $f = \left[g/2\pi\lambda \right]^{\frac{1}{2}}$. With $\omega = 2\pi f$ and $k = 2\pi/\lambda$

we get

$$\omega = 2\pi \left[\frac{g}{2\pi (2\pi/k)} \right]^{\frac{1}{2}} = [gk]^{\frac{1}{2}}$$

$$v_p = \frac{\omega}{k} = \frac{[gk]^{\frac{1}{2}}}{k} = \left[\frac{g}{k} \right]^{\frac{1}{2}} = \left[\frac{g\lambda}{2\pi} \right]^{\frac{1}{2}} \qquad \boxed{v_p = \left[\frac{g\lambda}{2\pi} \right]^{1/2}}$$

$$v_g = \frac{d\omega}{dk} = \frac{1}{2}\sqrt{g}\,(k)^{-\frac{1}{2}} = \frac{1}{2}\left[\frac{g}{k} \right] \Rightarrow \boxed{v_g = \frac{1}{2}\left[\frac{g\lambda}{2\pi} \right]^{\frac{1}{2}}}$$

5-27) From the energy time uncertainty principle $\Delta E \gtrsim \hbar/\tau$

$$\Delta E = \frac{1.054\times10^{-34}\,J.s}{10^{-7}s} = \boxed{1.054\times10^{-27}\,J = 6.58\times10^{-9}\,eV}$$

5-28) If the uncertainty in the speed is $\Delta v = 0.01\,cm/s$ then

$$\Delta p = m\Delta v = 10^{-6}g \cdot 10^{-2}cm/s = 10^{-9}\,kg \cdot 10^{-4}\,m/s$$

$$\Delta p = 10^{-13}\,kg.m/s$$

Then

$$\Delta x \sim h/\Delta p \sim \frac{6.626\times10^{-34}\,J.s}{10^{-13}\,kg.m/s} = \boxed{6\times10^{-21}\,m}$$

5-36) We have $\Delta x \sim 2\times10^{-15}\,m$ so $\Delta p \sim h/\Delta x = \dfrac{6.626\times10^{-34}\,J/s}{2\times10^{-15}\,m}$

$$= 3.3\times10^{-19}\,kg.m/s$$

So the typical p might be around half this value.

$$p \sim 1.6\times10^{-19}\,kg\,m/s$$

$$pc \sim (1.6\times10^{-19}\,kg\,m/s)(3\times10^{8}\,m/s) = 5\times10^{-11}\,J$$

$$\sim 3\times10^{8}\,eV$$

$E = (pc)^2/2mc^2 \simeq 40$ MeV for neutrons

$E = \quad$ " $\quad \simeq 8\times10^{4}$ MeV for electrons (using the non-relativistic formula)